CIS 525 Software Development of Parallel and Distributed Systems

Siphons, Traps, Liveness, Deadlock-Freeness

in Petri Nets

Let us assume that N=(P, T, F) is an ordinary Petri net. A siphon is a subset Σ of P (places) such that the set of its input transitions is contained in the set of its output transitions, i.e.

subset Σ of P is a **siphon** iff $^{*}\Sigma \subseteq \Sigma^{*}$

subset Θ of P is a **trap** iff $\Theta^{*} \subseteq {}^{*}\Theta$

Example: Consider a net N= (P, T, F), as presented in a Fig. 1 below, such that P={p_1, p_2, p_3, p_4, p_5, p_6, p_7} and let us consider Σ ⊆ P, where Σ={p_1, p_2, p_4, p_5, p_6}.

Input transitions of set Σ are $^{*}\Sigma$={t_7, t_1, t_2, t_3, t_5}

Output transitions of set Σ are Σ^{*}={t_1, t_2, t_3, t_5, t_6, t^7}

Because $\Sigma^{*} = {}^{*}\Sigma \cup \{t_6\}$ therefore Σ is a **siphon**. The siphon Σ contains a trap because Θ=Σ−{p5}.

Lets do the calculations: Θ = {p_1, p_2, p_4, p_6} = Σ-{p5}. The set of output transitions and input transitions are the same and are equal: {t_1, t_2, t_3, t_5, t_7} = $\Theta^{*}={}^{*}\Theta$. This means that Θ is a trap.

Siphons and traps are reverse concepts. A subset of places of a net N is a **siphon** iff it is a **trap** on the reverse net, N^{-1} (i.e. net obtained by reversing the arcs and its flow relation). Structural deadlocks are precisely siphons (by analogy to "soda siphons").

Properties:

1. If $m \in RS(N, m_0)$ is a deadlock state then
   $\Sigma=\{p: m(p)=0\}$ is an unmarked (empty) siphon

2. If a siphon is (or becomes) unmarked then it will remain unmarked for any possible net system evolution. Therefore all its input and output transitions are dead. So the system is not live (but it can be deadlock-free).

3. If a trap is (or becomes) marked, it will remain marked for any possible net system evolution (i.e. at least one token is trapped).

Example: In this example we are showing a difference between non-liveness and deadlock-freeness. This example of Fig.15.b is taken from the textbook. Fig. 15.b presents a net that is:

a) **Non-live** – it contains a sequence of transitions from initial marking m_0 such that transitions c and d are not executed at all
b) **Deadlock-free** – after initial execution of transition c followed by d a sequence $(ab)^+$ is executed indefinitely.

(N, m_0) is non-live and it is also deadlock-free (it continues to execute a sequence of $(ab)^+$ after initial sequence of cd.

Illustrations of concepts of siphons and traps:

a) **Siphon –** tokens arrive to p and they are siphoned out of p because there is more output transitions than input transitions
b) **Trap –** tokens arriving to p are trapped in p because number of output transitions is smaller than input transitions

Corollaries:

1. Liveness implies deadlock-freeness.
2. Deadlock freeness does not imply liveness (see example on previous page).

The MST property

Definition 1: Let N be an arbitrary net. The system (N,m_0) has the marked-siphon-trap (MST) property if each siphon contains a marked trap at m_0.

Definition 2: A siphon (trap) is minimal if it does not contain another siphon (trap).

Thus siphons in the above statement can be constrained to be minimal without any loss of generality. The MST property guarantees that all siphons will be marked. Thus no dead marking can be reached.

Theorem: If (N,m_0) has the MST property then the system is deadlock-free.

Example: Fig. 15.a from the textbook.

Observations:

1. MST property does not hold.
2. Net is live and bounded.