NET MORPHISMS

1. MOTIVATION:

- homogeneous formalism to represent models of different layers (relations between models can be expressed in a uniform way)

- examples of ***relations between models:***

  - abstractions from details
  -composition of subcomponents (properties of subcomponents deduced from respective properties of the whole system and properties of subparts carry over to the composed system)

- ***Petri nets support****:* causality, choice (conflicts, and concurrency)

- ***S-elements*** and ***T-elements*** are dual to each other as choice and concurrency are dual concepts

- ***S-components*** and ***T-components*** (S-component is a subnet where only S-elements are branched)

- ***computation of S-components*** or covering of a net by S-components is a highly complex task

- well-designed system can be modeled as a ***composition*** of simple subparts

- ***transformations of nets*** which preserve certain properties

- ***net morphisms*:** a method of relating nets N and N': a mapping from elements of a source net to elements of a target net which respects the bipartision and the flow relation of the source net

- ***vicinity respecting morphisms***: if two elements are similar enough to be mapped to the same element, then their effect on the environment should be the same as well (effect on environment - adjacent elements)

- ***structure and semantics are not independent*** (net covered by S-components is bounded independetly of the marking class considered - structural property preserved by morphisms).

2. DEFINITIONS:

- definition of a net N=(S,T,F)

- some notation: $^{\bullet}x$, $x^{\bullet}$, $^{\circ}x$, x°

- subnet, S-subnet, T-subnet

- adjacency relation of a net $P = (F \cup F^{-1}) \cap (SxT)$

- **net morphism:**

Let N an N' be nets. A mapping f: X --->X' is called net morphism, denoted by f: N --->N' if for every a,b in X the following properties hold:

$$1.\ (a,b) \in P \Rightarrow (f(a), f(b)) \in P' \cup id_{X'}$$

$$2.\ (a,b) \in F \Rightarrow (f(a), f(b)) \in F' \cup id_{X'}$$

where, $id_{X'} = \{(x,x) : x \in X'\}$.

Theorem: Let f:N--->N' be a **net morphism**. Then

1. $\forall_{t \in T} \forall_{s \in S'} \; f(t) = s \Rightarrow f({}^{\bullet}t \cup \{t\} \cup t^{\bullet}) = \{s\}$

2. $\forall_{s \in S} \forall_{t \in T'} \; f(s) = t \Rightarrow f({}^{\bullet}s \cup \{s\} \cup s^{\bullet}) = \{t\}$

Morphisms respect the flow relation of the source net. Sequences of arcs are also respected in a certain sense.

Morphisms are **surjective** mappings (net N is a more detailed model of a system than N').

Composing nets is a special case of identifying elements.

Transformations or generation of nets (only source net and some information about the mapping is given; target net is given only implicitly; target net *is **generated by the morphism***).

Given the partition of the elements which is generated by the equivalence relation **'is mapped to the same element'**, the generated target net is unique up to isomorphism. These morphisms are called **quotients.**

A **surjective net morphism** f: N --->N' is called **quotient** if

$$\forall (x, y) \in F' \; \exists (a, b) \in F \text{ with } f(a) = x \text{ and } f(b) = y$$

Remark:

Only few properties are transferred from a source net to a target net by a morphism and even less by a quotient.

Vicinity respecting morphisms

- the pre-set of each element i mapped surjectively to the pre-set of its image, i.e. $f({}^{\circ}a) \subseteq {}^{\circ}f(a)$ and $f(a^{\circ}) \subseteq (f(a))^{\circ}$ for each element a

(this approach means that S-elements are mapped into S-elements and T-elements into T-elements)

Fig.4 - ***'line-reducing morphisms'*** do not respect vicinities in this sense.

- we want to keep the possibility of mapping an element together with (a part of) its pre-set and (a part of) its post-set to one element of the target net

- **A net morphism** f:N---->N' is said to be S-vicinity respecting if $\forall a \in S$:

1. $f({}^{\circ}a) \subseteq \{f(a)\} \vee f({}^{\circ}a) \subseteq {}^{\circ}f(a)$
2. $f(a^{\circ}) \subseteq \{f(a)\} \vee f(a^{\circ}) \subseteq f(a)^{\circ}$

Fig.4 - S-vicinity respecting morphisms.