Comparison of TSP Algorithms

Project for
Models in Facilities Planning and
Materials Handling

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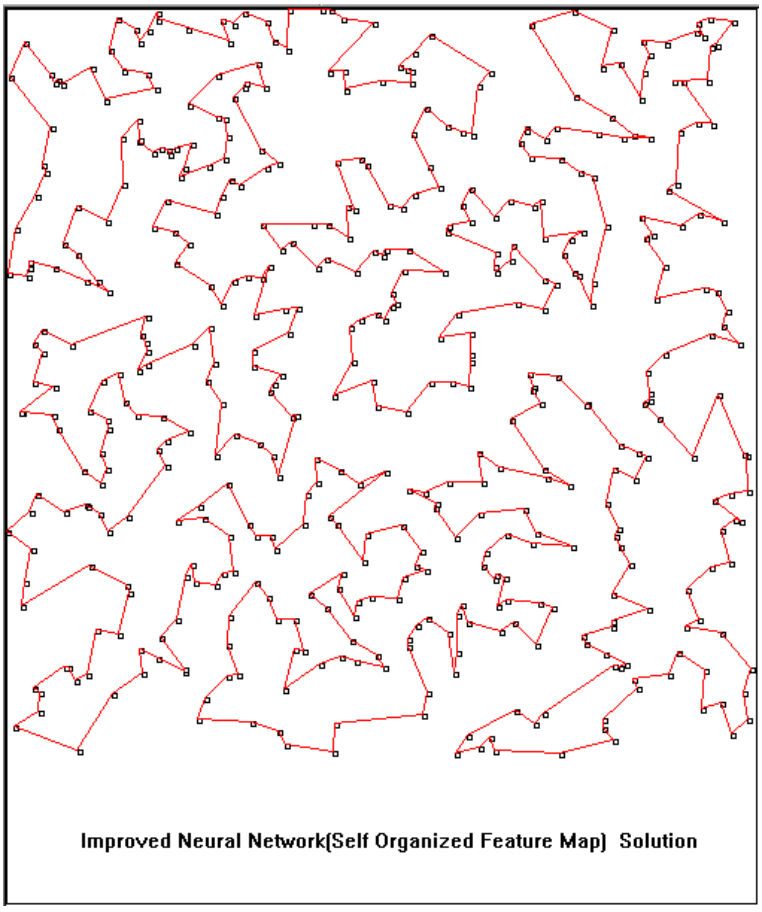
Executive Summary

Our purpose in this term project is to implement heuristic algorithms and compare and evaluate their respective computational efficiency. Included in this model are greedy, 2-opt, greedy 2-opt, 3-opt, greedy 3-opt, genetic algorithm, simulated annealing, and neural network approach and their improvement versions. The problem size can be adjusted from 2-node up to 10,000-node. Therefore, these algorithms can be evaluated in a wide spectrum of situations.

We implemented above heuristic algorithms. The conclusion that is derived from this project is for small-size TSP ($n \leq 50$), greedy 2-opt algorithm performs pretty well, i.e high optimality vs. little computational time. The neural network(Self Organizing feature Maps) algorithm shows the best efficiency for all city sizes.

Furthermore the algorithms were improved by using non crossing methods. This improved methods always result in better solutions



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1. Introduction

The Travelling Salesman Problem (TSP) is a deceptively simple combinatorial problem. It can be stated very simply: A salesman spends his time visiting N cities (or nodes) cyclically. In one tour he visits each city just once, and finishes up where he started. In what order should he visit them to minimize the distance traveled? TSP is applied in many different places such as warehousing, material handling and facility planning.

Although optimal algorithms exist for solving the TSP, like the IP formulation, industrial engineers have realized that it is computationally infeasible to obtain the optimal solution to TSP. And it has been proved that for large-size TSP, it is almost impossible to generate an optimal solution within a reasonable amount of time. Heuristics, instead of optimal algorithms, are extensively used to solved such problems. People conceived many heuristic algorithms to get near-optimal solutions. Among them, there are greedy, 2-opt, 3-opt, simulated annealing, genetic algorithms, neural network approach, etc. But their efficiencies vary from case to case, from size to size.

The algorithms can be implemented by using Visual C++. From these program the results are attained and the results of each algorithms can be compared. These algorithms can be improved by using the non-crossing method which is explained in "Improvement Algorithm". The comparison of the results shows the optimality of each algorithm varies with the problem size.

2. Description of Algorithms

The heuristic algorithms for TSP can be classified as

1. Construction Algorithms;
2. Improvement Algorithms; and
3. Hybrid Algorithms.

Construction algorithms are those in which the tour is constructed by including points in the tours, usually one at a time, until a complete tour is developed. In improvement algorithms, a given initial solution is improved, if possible, by transposing two or more points in the initial tour. With regard to improvement algorithms, two strategies are possible. We may either consider all possible exchanges and choose the one that results in the greatest savings and continue this process until no further reduction is possible; or as soon as a savings is available, we may make the exchange and examine other possible exchanges and keep the process until we cannot improve the solutions any further. In our project, the former are 2-opt and 3-opt algorithms; and the latter are greedy 2-opt and greedy 3-opt algorithms. Hybrid algorithms use a construction algorithms to obtain an initial solution and then improve it using an improvement algorithm. In our project, simulated annealing and neural network fall into this category.

A. Greedy Algorithm

Greedy algorithm is the simplest improvement algorithm. It starts with the departure Node 1. Then the algorithm calculates all the distances to other n-1 nodes. Go to the next closest node. Take the current node as the departing node, and select the next nearest node from the remaining n-2 nodes. The process continues until all the nodes are visited once and only once then back to Node 1. When the algorithm is terminated, the sequence is returned as the best tour; and its associated OFV is the final solution. The advantage of this algorithm is its simplicity to understand and implement. Sometime, it may lead to a very good solution when the problem size is small. Also, because this algorithm does not entail any exchange of nodes, it saves considerably much computational time.

For the C++ code for greedy algorithm, please refer to the Appendix.

B. 2-Opt Algorithm

For n nodes in the TSP problem, the 2-Opt algorithm consists of three steps:

Step 1 Let S be the initial solution provided by the user and z its objective function value (In our model, we use greedy algorithm to setup the initial solution and objective function value.). Set $S^*=s$, $z^*=z$, $i=1$ and $j=i+1=2$.

Step 2 Consider the exchange results in a solution S' that has OFV $z'<z^*$, set $z^*=z'$ and $S^*=S'$. If $j<n$ repeat step 2; otherwise set $i=i+1$ and $j=i+1$. If $i<n$, repeat step 2; otherwise go to step 3.

Step 3 If $S \neq S^*$, set $S=S^*$, $z=z^*$, $i=1$, $j=i+1=2$ and go to step 2. Otherwise, output S^* as the best solution and terminate the process.

Observe that the 2-opt algorithm considers only pairwise exchange. Initially, the algorithm considers transposition of Nodes 1 and 2. If the resulting solution's OFV is smaller than that of the initial solution, it is stored as a candidate for future consideration. If not, it is discarded and the algorithm considers transposing of Nodes 1 and 3. If this exchange generates a better solution, it is stored as a candidate for future consideration; if not, it is discarded. Thus, whenever a better solution is found, the algorithm discards the previous best solution. This procedure continues until all the pairwise exchanges are considered. Because each node can be exchanged with n-1 other nodes and there are n nodes in total, there are n(n-1)/2 different exchanges. These n(n-1)/2 exchanges are considered in step 2. The solution retained at the end of step 2 is the one that provides the most improvement in the OFV. Starting with this as the new solution, the algorithm repeats step 2 to find another better solution. At some stage, no improvement in the current best solution is possible, and then the algorithm terminates. The remaining solution is returned to the user as the best one.

For the C++ code for 2-opt algorithm, please refer to the Appendix.

C. Greedy 2-Opt Algorithm

The greedy 2-Opt algorithm is a variant of 2-opt algorithm. It also consists of three steps:

Step 1 Let S be the initial solution provided by the user and z its objective function value (In our model, we use greedy algorithm to setup the initial solution and objective function value.). Set S*=s, z*=z, i=1 and j=i+1=2.

Step 2 Transpose Node i and Node j, i<j. Compare the resulting OFV z with z*. If $z \geq z^*$ and j<n, set j=j+1 and repeat step 2; Otherwise go to step 3.

Step 3 If z<z*, set S*=S, z*=z, i=1, j=i+1=2 and go to step 2. If $z \geq z^*$ and j=n, set i=i+1, j=j+1 and repeat step 2. Otherwise, output S* as the best solution and terminate the process.

Like the 2-Opt algorithm, greedy 2-opt algorithm also considers pairwise exchanges. Initially, it considers transposing Nodes 1 and 2. If the resulting OFV is less than the previous one, two nodes are immediately transposed. If not, the algorithm will go on to Node 3 and evaluate the exchange, and so on until find the improvement. If Nodes 1 and 2 are transposed, the algorithm will take it as an initial solution and repeat the algorithm until it is impossible to improve the solution any further. Greedy 2-opt algorithm makes the exchange permanent whenever an improvement is found and thus consumes less computational time than 2-Opt algorithm. On the other hand, greedy 2-opt produces slightly worse solutions than 2-Opt algorithm.

For the C++ code for greedy 2-opt algorithm, please refer to the Appendix.

D. 3-Opt Algorithm

The 3-Opt algorithm is similar to the 2-opt algorithm except that it considers transposing two nodes at a time. There are two possible ways of transposition.: i->j->k->i
and j->k->i->j. Let's just consider the first transposition. The steps involved are

Step 1. Let S be the initial solution and z its OFV. Set S*=S,z*=z,i=1,j=i+1 and k=j+1.

Step 2. Consider transposing Nodes i, j, k in "i->j->k->i" fashion. If the resulting solution S' has a smaller z'<z*, set z*=z' and S*=S'. If k<n, set k=k+1. Otherwise, set j=j+1. If j<n-1, set k=j+1. Otherwise, set i=i+1, j=j+1 and k=j+1. If i<n-2, repeat step 2. Otherwise go to step 3.

Step 3 If $S \neq S^*$, set S=S*, z=z*, i=1, j=i+1,k=j+1 and go to step 2. Otherwise, output S* as the best solution and terminate the process.

For the C++ code for 3-Opt algorithm, please refer to the Appendix.

E. Greedy 3-Opt Algorithm

Like the greedy 2-opt algorithm, greedy 3-opt algorithm also makes the 3-node exchange permanent whenever its resulting OFV is better than the current

OFV and repeats the algorithm with the new transposition as the initial solution. Let's just consider the "i->j->k->i" transposition again. The steps involved are:

Step 1 Let S be the initial solution provided by the user and z its objective function value (In our model, we use greedy algorithm to setup the initial solution and objective function value.). Set S*=S,z*=z,i=1,j=i+1 and k=j+1.

Step 2. Consider transposing Nodes i, j, k in 'i->j->k->i" fashion. If the resulting solution S' has a smaller z'<z*, set z*=z' and S*=S'. If k<n, set k=k+1. Otherwise, set j=j+1. If j<n-1, set k=j+1. Otherwise, set i=i+1, j=j+1 and k=j+1. If i<n-2, repeat step 2. Otherwise go to step 3.

Step 3 If z<z*, set S*=S, z*=z, i=1, j=i+1=2, k=j+1 and go to step 2. If $z \geq z^*$ and k=n, set i=i+1, j=j+1, and repeat step 2. . If $z \geq z^*$ and j=n-1, set i=i+1, and repeat step 2. Otherwise, output S* as the best solution and terminate the process

The advantage of greedy 3-Opt algorithm over 3-Opt algorithm is once again less computational time due to its less thorough search in the network.

For the C++ code for greedy 3-Opt algorithm, please refer to the Appendix.

F. Genetic Algorithm

As the name implies, this algorithm gets its idea from genetics. The algorithm works this way.

Step 0 Obtain the maximum number of individuals in the population P and the maximum number of generations G from the user, generate P solutions for the first generation's population randomly, and represent each solution as a string. Set generation counter N_{gen}=1.

Step 1 Determine the fitness of each solution in the current generation's population and record the string that has the best fitness.

Step 2 Generate solutions for the next generation's population as follows:

1. Retain 0.1P of the solutions with the best fitness in the previous population.
2. Generate 0.89P solutions via mating, and
3. Select 0.01P solutions from the previous population randomly and mutate them.

Step 3 Update N_{gen}= N_{gen}+1. If $N_{gen} \leq G$, go to Step 1. Otherwise stop.

In our project, we used the two-point crossover method, given two parent chromosomes $\{x_1,x_2,\ldots x_n\}$ and $\{y_1,y_2,\ldots,y_n\}$, two integers r and s are randomly selected, such that $1 \leq r \leq s \leq n$. And the genes in positions r to s of one parent are swapped with those of the other to get two offsprings as follows.

$\{x_1, x_2,\ldots, x_{r-1}, y_r, y_{r+1},\ldots, y_s, x_{s+1}, x_{s+2},\ldots, x_n\}$

$\{y_1, y_2,\ldots, y_{r-1}, x_r, x_{r+1},\ldots, x_s, y_{s+1}, y_{s+2},\ldots, y_n\}$

Either both or the better of the two offspring is then included in the new population. This mutation procedure is repeated until the required number of offspring is generated. To avoid generating infeasible solutions, we also used the

partially matched crossover method. Set t=r, and swaps genes x_t and y_t if $x_t \neq y_t$ in the first parent $x=\{x_1, x_2,\ldots, x_{r-1}, x_r, x_{r+1},\ldots, x_s, x_{s+1}, x_{s+2},\ldots, x_n\}$. This swapping is done for t=r through s, one by one, and then we have a feasible offspring. The same is done for $y=\{y_1, y_2,\ldots, y_{r-1}, y_r, y_{r+1},\ldots, y_s, y_{s+1}, y_{s+2},\ldots, y_n\}$. Because the partially matched crossover method always swaps genes within a parent, feasible offspring are obtained.

Therefore, our model uses mutation to generate a population by swapping any two randomly selected genes.

For the C++ code for genetic algorithm, please refer to the Appendix.

H. **Simulated Annealing**

The basic idea of simulated annealing(SA) is from the statistical mechanics and is motivated by an analogy of behavior of physical systems in the presents of a heat bath. This algorithm obtains better final solutions by gradually going from one solution to the next. The main difference of SA from the 2-opt or 3-opt algorithm is that the local optimization algorithm is often restrict their search for the optimal solution in a downhill direction which mean that the initial solution is changed only if it results in a decrease in the OFV. The temperature refers to the state through which the simulated annealing algorithm passes in its search for a better solution. Starting with an initial temperature, we move to the next temperature only when we have reached a frozen state. When a frozen state is reached, the temperature is reduced by a cooling factor r, and the procedure is repeated until a certain predetermined number of temperature steps have been performed.

Step 0 Set *S* = initial feasible solution
z = corresponding OFV
T = initial temperature = 1*; r* = cooling factor = 0.9
ITEMP = number of times the temperature *T* is decreased = 0
NLIMIT = maximum number of new solutions to be accepted at each temperature = 10n
NOVER = maximum number of solutions evaluated at each temperature = 100n

Step 1 Repeat step 2 *NOVER* times or until the number of successful new solutions is equal to *NLIMIT*

Step2 Pick a pair of machines randomly and exchange their positions. If the exchange of the position of the two machines results in the overlapping of some other pairs of cities, appropriately modify the coordinates of the center of the concerned machines to ensure no overlapping. If the resulting solution S* has OFV <= *z*, set S* = S and *z* = corresponding OFV. Otherwise, compute *delta* = difference between z and the OFV of the solution S*. and set S*=S with a probability $e^{-delta/T}$.

Step 3 Set *T* = *rT* and *ITEMP* = *ITEMP* + *1*. If *ITEMP* <= 100, go to step 1; otherwise stop.

For the C++ code for simulated annealing algorithm, please refer to the Appendix.

G. Neural Network(Self Organizing Feature Maps)

First, randomly pick up any city in the network as the initial loop. The number of nodes N on the loop grows subsequently according to a node creation process. (see below). At each processing step, a city i is surveyed. One complete iteration takes M (number of total cities to be visited) sequential steps, for i=1 to i=M, thus picking every city once in a preset order. A gain parameter is used which decreases between two complete iterations. Several iterations are needed to go from the high gain value to the low one giving the final result.

Surveying the city i comprises the following steps:

Step 1. Find the node j_c which is closed to city i.

Step 2. Move node j_c and its neighbors on the ring towards city i. The distance each node will move is determined by a function f(G,n) where G is the gain parameter (its initial is set 10 in our model), and n is the distance measured along the loop between nodes j and j_c.

n=inf (j- j_c (mod N), j_c -j(mod N))

Every node j is moved from current position to a new one.

The function f is defined to be

$f(G,n)=sqrt(1/2)*exp(-n^2/G^2)$

This means:

- when G $\rightarrow\infty$, all nodes move towards city I with the same strength sqrt(1/2).
- when G $\rightarrow$ 0, only node jc moves towards city i.

Decreasing the gain at the end of a complete survey is done by G$\leftarrow$(1-α)G. α is the only parameter that has to be adjusted. It is directly related to the number of complete surveys needed to attain the result, and hence its quality.

The gain is decreased from a high initial G_i which ensures large moves for all nodes at each iteration step to a sufficiently low G_f for which the network is stabilized. The total number of iterations can be computed from these two fixed values depending only on M.

A node is duplicated, if it is chosen as the winner for two different cities in the same complete survey. The newly created node is inserted into the ring as a neighbor of the winner, and with the same coordinates in the plane. Both the winner and the created node are inhibited. If chose by a city, an inhibited node will induce no movement at all in the network for this city presentation. It is re-enabled on the next presentation. This guarantees that the “twin nodes” will be separated by the moves of their neighbors, before being “caught” by cities. The maximum number of nodes created on the loop experimentally appears to be less than 2M.

A node is deleted, if it has not been chosen as the winner by any city during three complete surveys. This creation-deletion mechanism has proved important to the attainment of near-optimum solutions.

For the C++ code for neural network approach, please refer to the Appendix.

3. Improvement of Algorithms

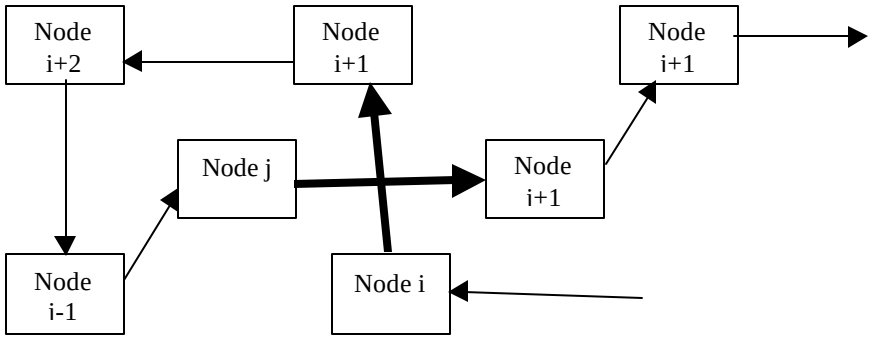
In the initial implementation of these 8 algorithms, it is found that these algorithms do not eliminate the crossing of routes. Of course, these crossings of routes lengthen the total distance of travel in TSP. We improved these algorithms by getting rid of these crossings.

The process is as follows.

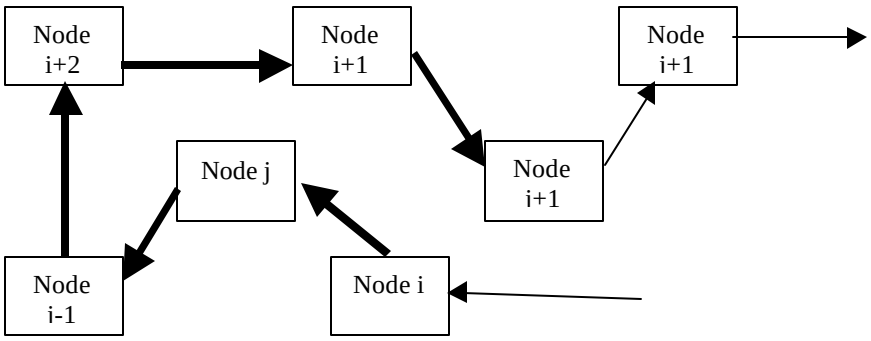
First, obtain the solution with any of the preceding algorithms. Second, pick up the segment between Node 1 and Node 2, check any other segment against the 1-2 segment to see if these 2 segments intersect each other. If not, pick up the segment 2-3, and check consequent segment against segment 2-3. If the intersection is found for any two distinct segments, re-route the loop. And the process is continued sequentially throughout the network.

The mathematical implementation of both checking and re-routing is as follows: The current 2 segments are $i \rightarrow i+1$, $j \mapsto j+1$ $(1 \le i < i+1 < j < j+1 \le n)$. The line of $i \rightarrow i+1$ segment can be expressed as $x=x_i+(x_{i-1} - x_i)*t$, $y= y_i +(y_{i+1} - y_i)*t$, and the line of $j \rightarrow j+1$ segment can be expressed as $x=x_j+(x_{j-1} - x_j)*s$, $y= y_j +(y_{j+1} - y_j)*s$. Given these parametric expressions and the coordinates of these 4 nodes, the coordinates and associated s, and t can be calculated for the intersection of these lines. If both s and t are
$0 \le t \le 1$ and $0 \le s \le 1$, it is determined that these two segments are intersected. Otherwise not. When they are intersected, transpose node i+1 and node j. There would be neither intersection on these 2 segments nor break of the loop.

Since there are N+1 routes for N cities, N(N+1)/2 crossing-checks are necessary. Our model finds these non-crossing methods substantially improved upon these algorithms for large-size problem.



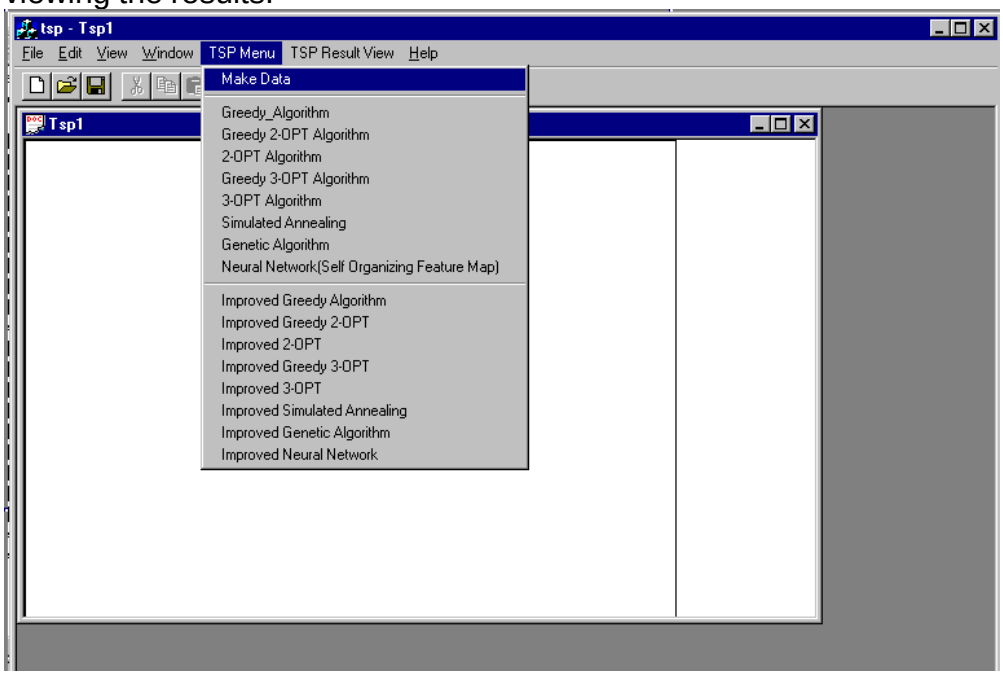
(Original Result)

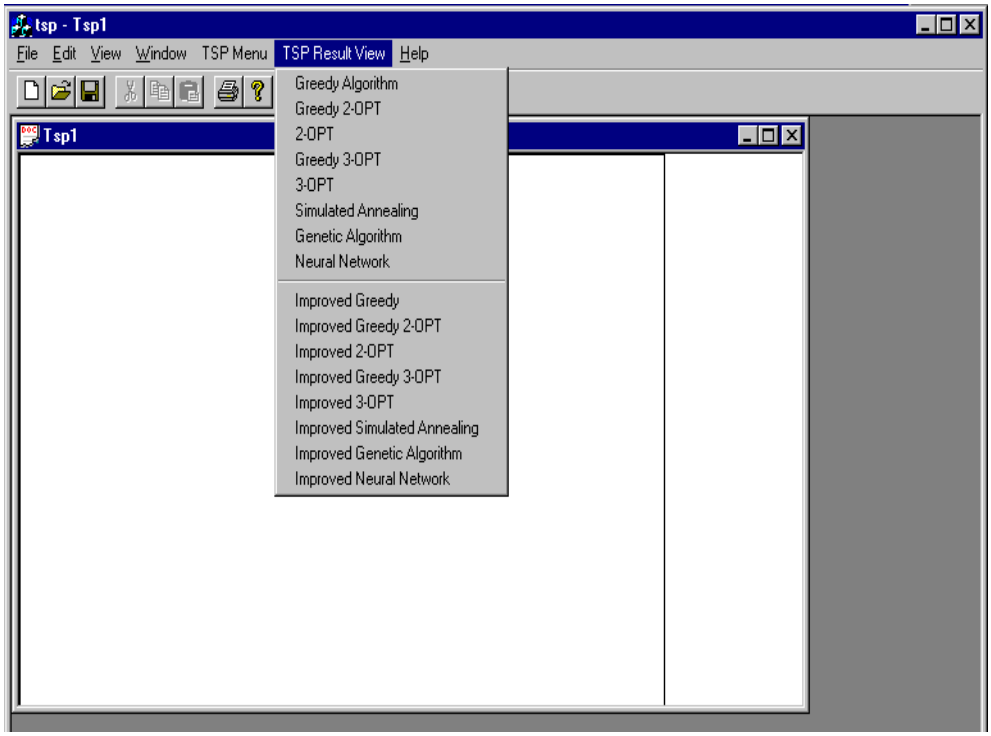


(Improved Result)

4. Implementation & Analysis

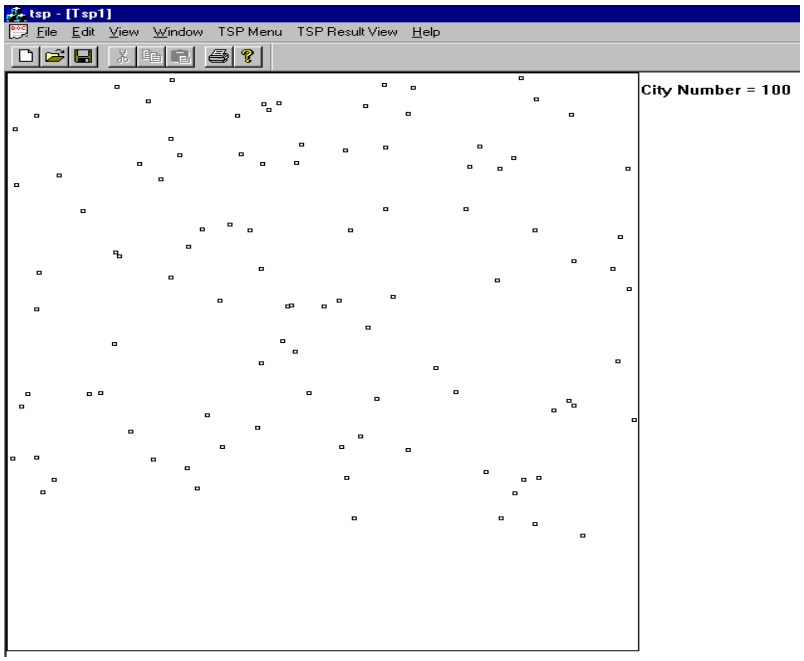
We implemented above algorithms by using Microsoft Visual C++ 5.0 (with MFC library). The main menu and windows are followed. <TSP menu> is menu for solving TSP by using several algorithms and <TSP Result View> is menu for viewing the results.



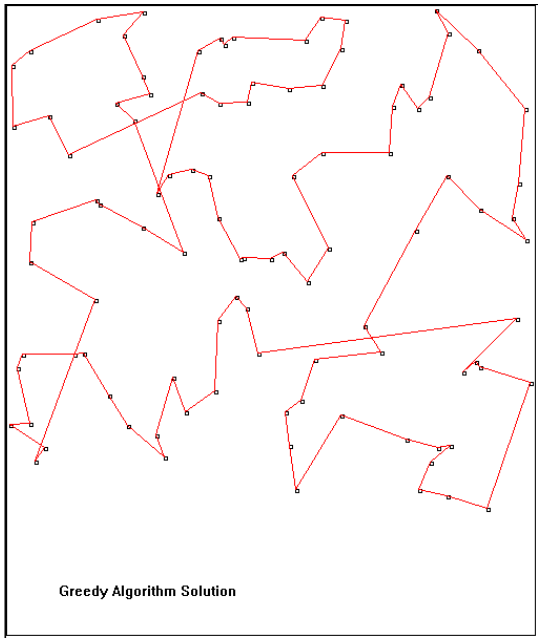


1) make Data

Our program can get the city number from user and make cities' locations by using random generation function. We fixed the maximum X and Y values to 500.



2) Greedy algorithm and Improved Greedy algorithm



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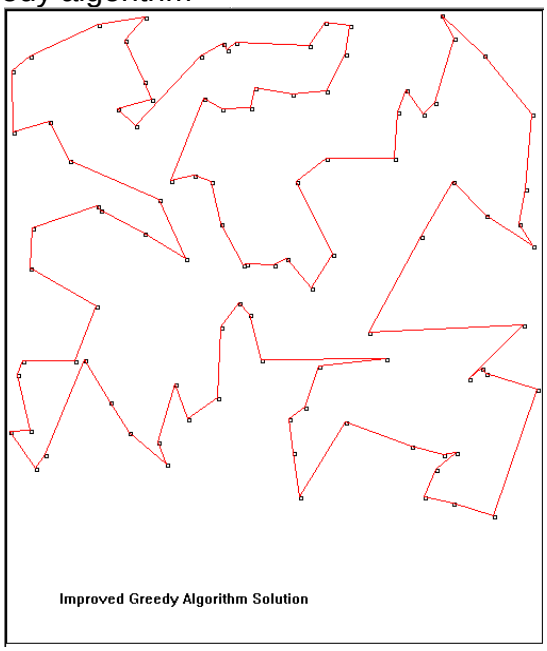

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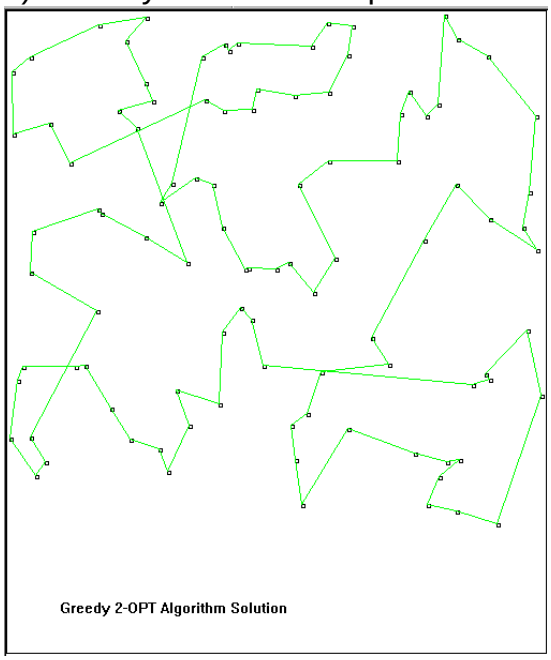
3) Greedy 2-OPT and Improved Greedy 2-OPT



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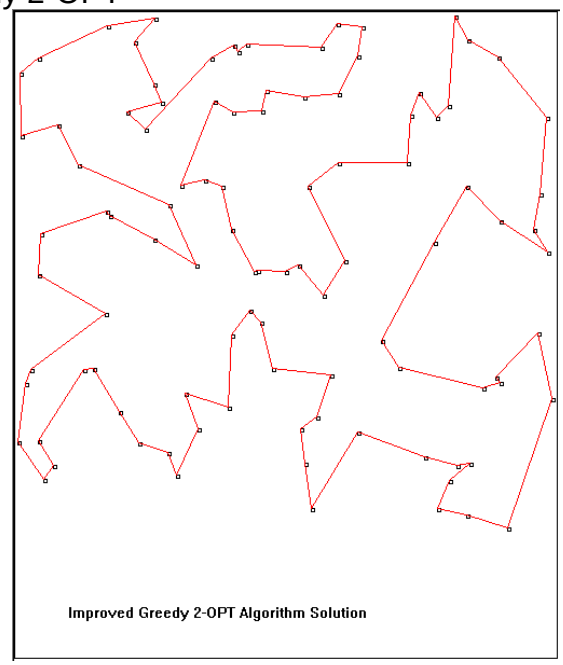

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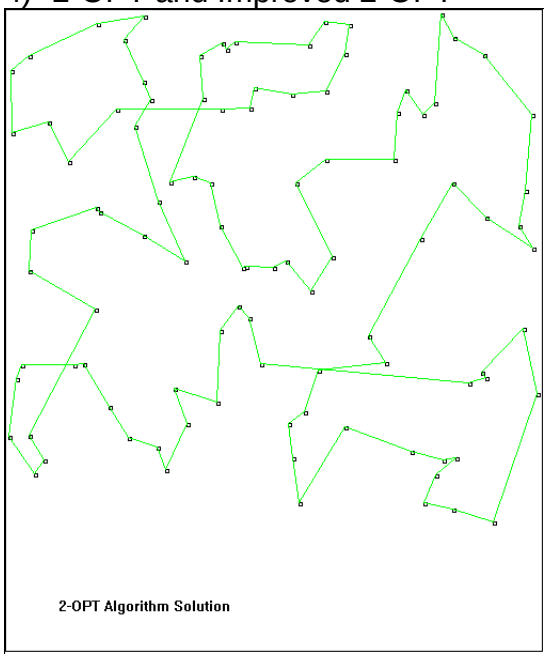
4) 2-OPT and Improved 2-OPT



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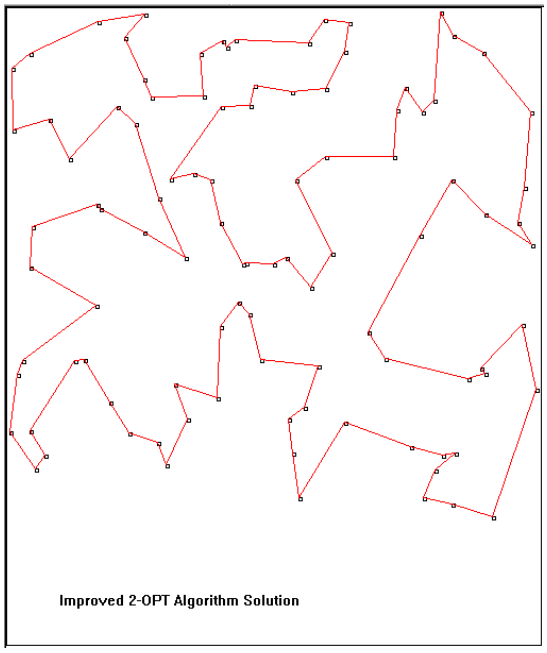

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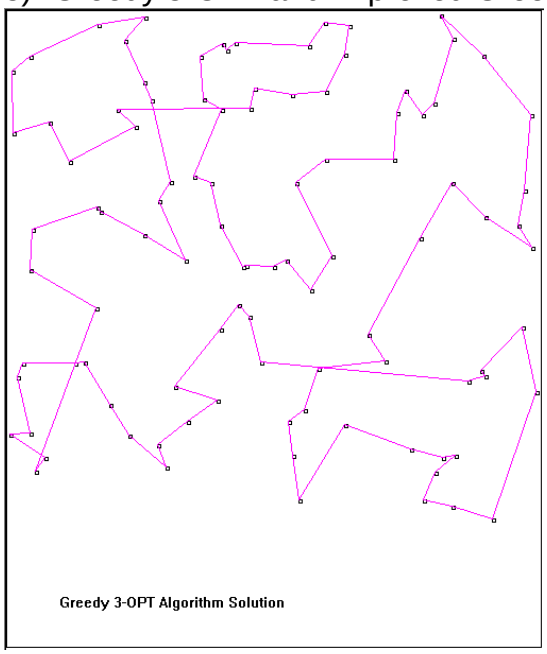

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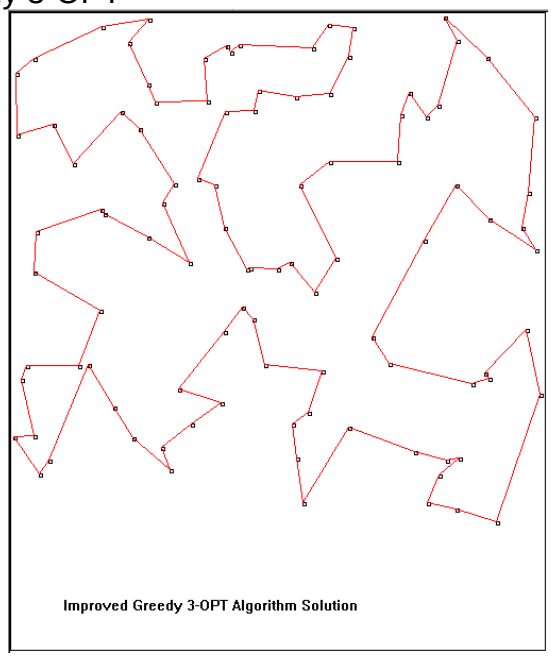

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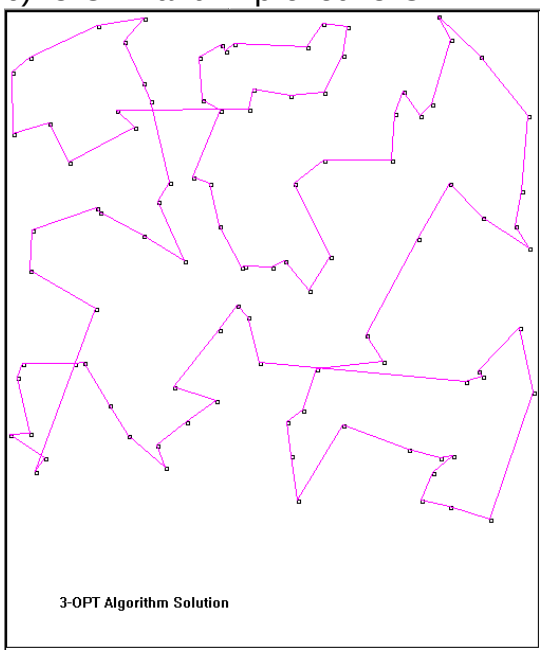
6) 3-OPT and Improved 3-OPT



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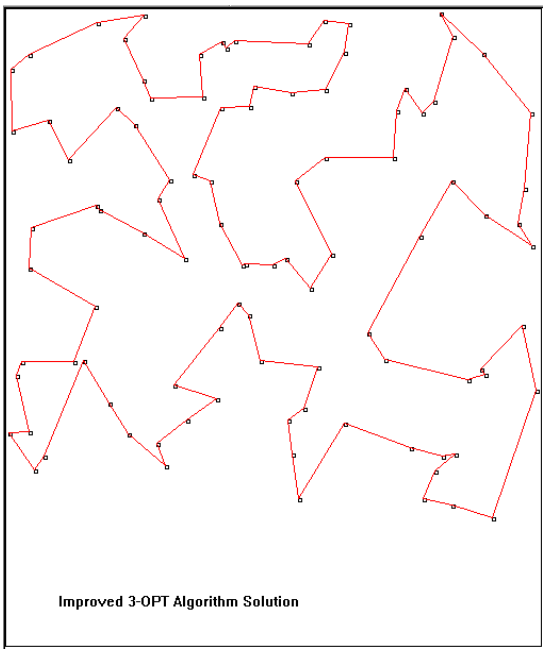

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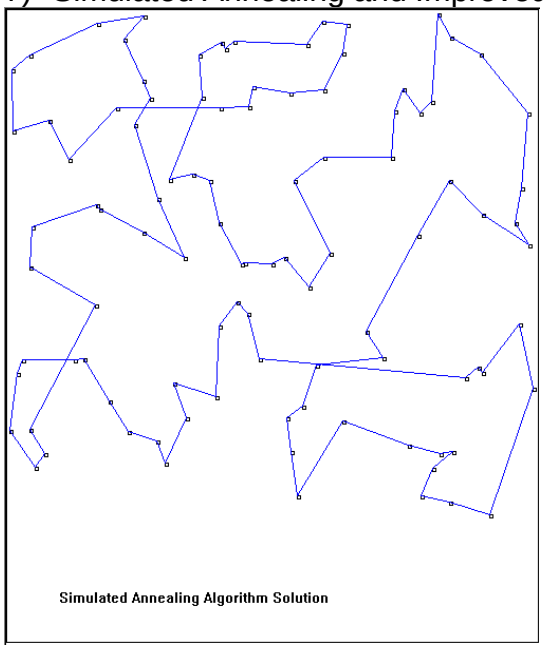
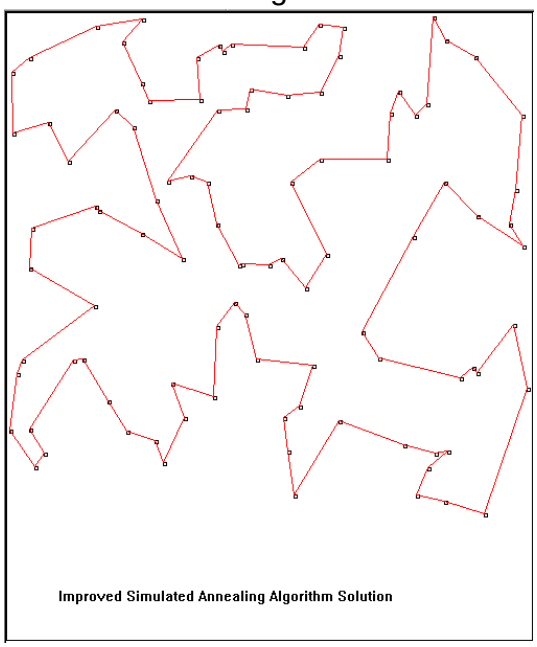
7) Simulated Annealing and Improved Simulated Annealing





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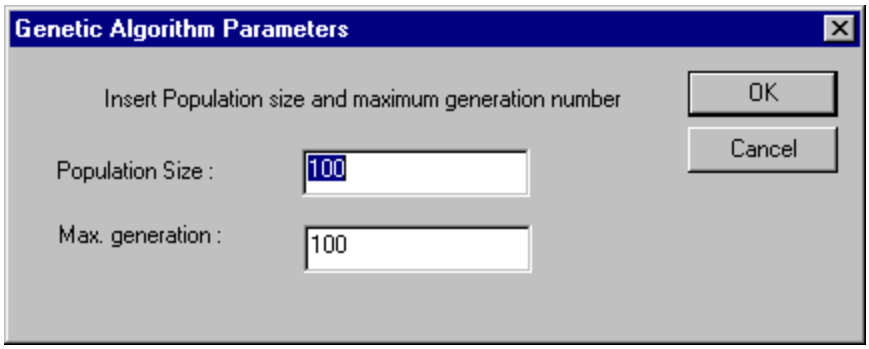
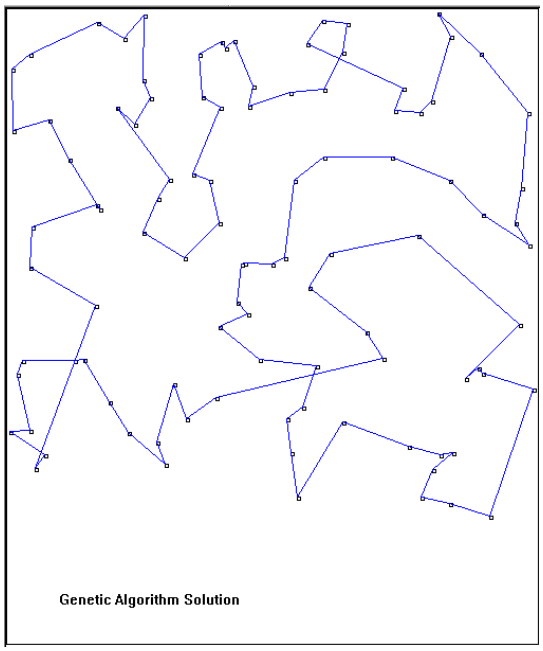
8) Genetic Algorithm and Improved Genetic Algorithm
The user can give Population size and generation size like following dialog



Following results used population size 100 and generation size 100



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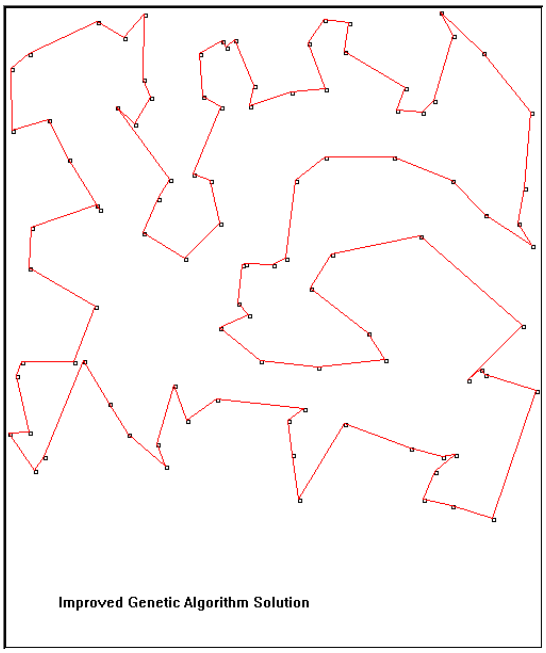

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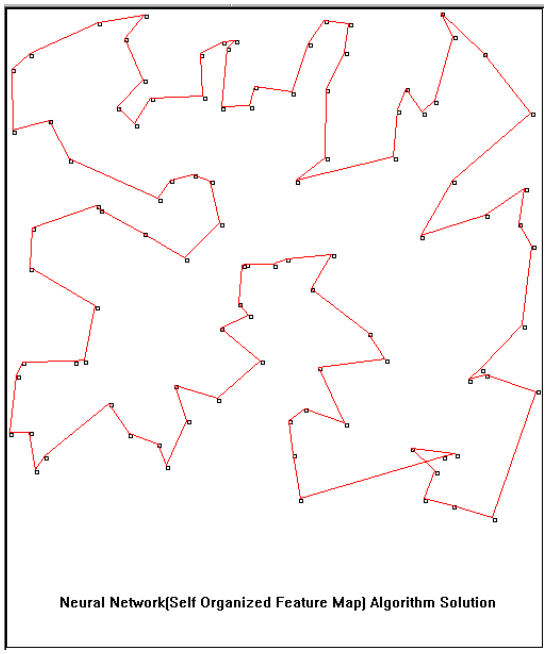
9) Neural Network(Self Organized Feature Map) and Improved Neural Network



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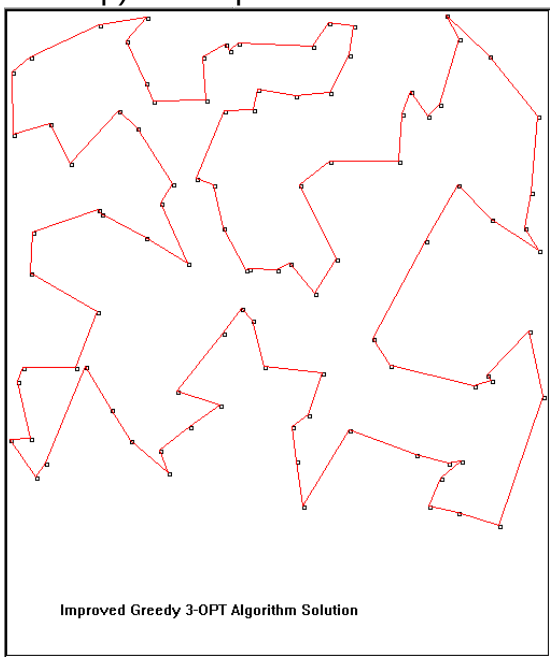
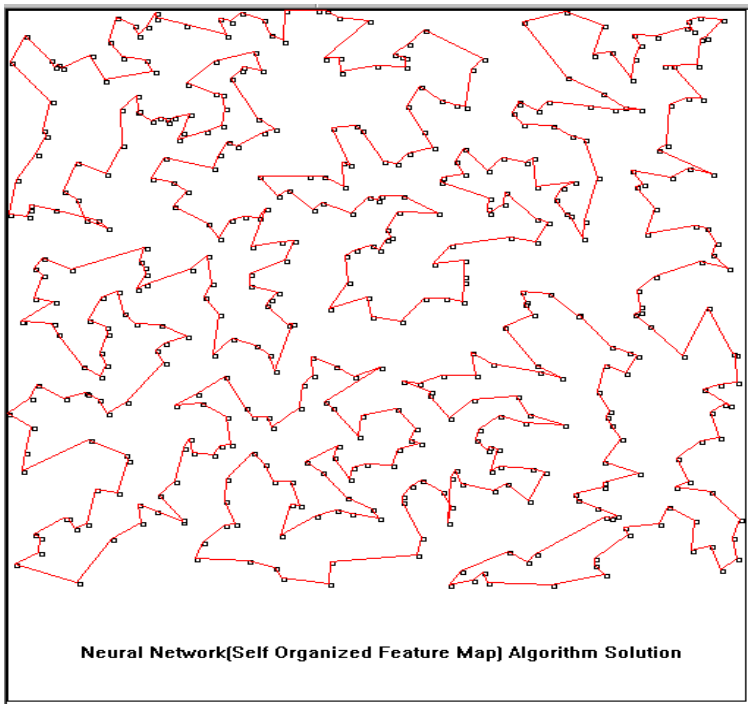
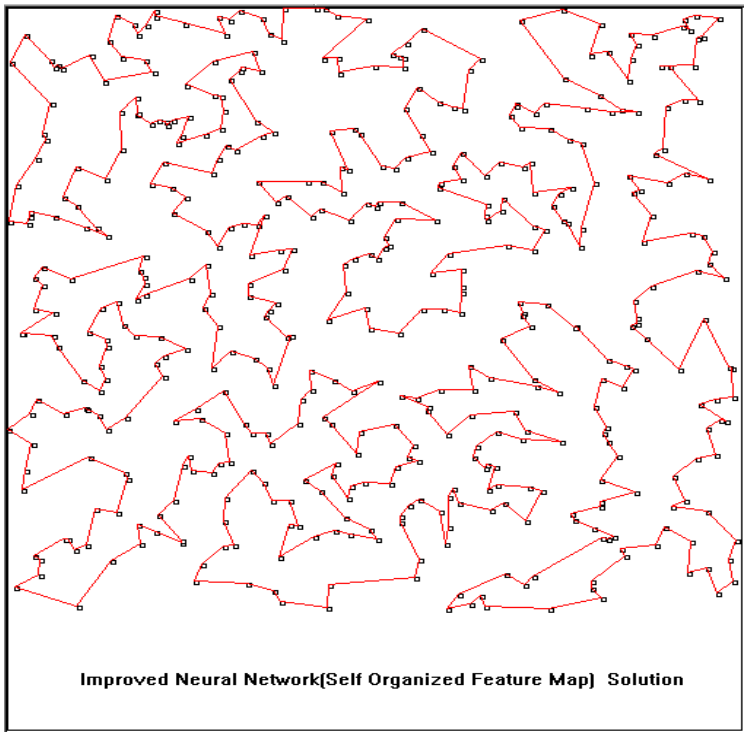


Length : 43995.19

500 cities case



Length : 8778.29
Time : 346 sec



Length : 8773.85

10) Analysis

We did a performance comparison of 8 algorithms and improved versions. For the simulation city size of 30, 100 and 500 were used. The below tables and graphs illustrate the result of simulation. The neural network algorithm came up with the shortest distance for all the number of cities. The improved version of the algorithms always resulted in better solution compared to the original solution. For the time of simulation, 3-Opt and Greedy 3-Opt had the longest time to find the solution. That is why for 500 cities, these algorithms did not give the solution. As the number of the cites increased the simulation time also increased for some of the algorithms. Excluding the 3-Opt and Greedy 3-Opt, the neural network took the longest time to simulate for 500 cites which was 346 seconds.

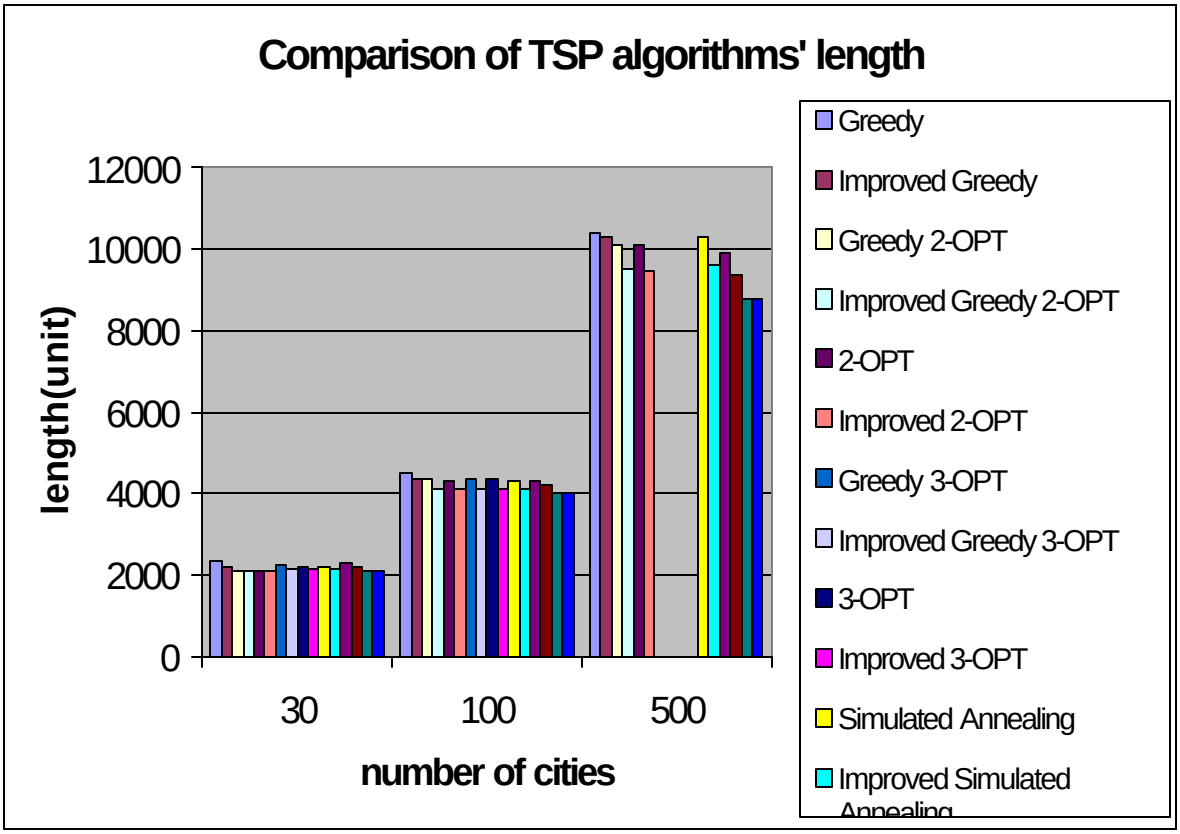
< Length Comparison >

Algorithm \ City Size	**30**	**100**	**500**
Greedy	2330	4478	10402
Improved Greedy	2187	4335	10279
Greedy 2-OPT	2095	4377	10127
Improved Greedy 2-OPT	2095	4128	9500
2-OPT	2095	4320	10083
Improved 2-OPT	2095	4122	9459
Greedy 3-OPT	2240	4356	
Improved Greedy 3-OPT	2168	4124	
3-OPT	2178	4356	
Improved 3-OPT	2155	4124	
Simulated Annealing	2177	4320	10279
Improved Simulated Annealing	2131	4122	9606
Genetic Algorithm	2295	4302	9906
Improved Genetic Algorithm	2201	4200	9364
Neural Network	2086	4035	8778
Improved Neural Network	2086	3995	8773

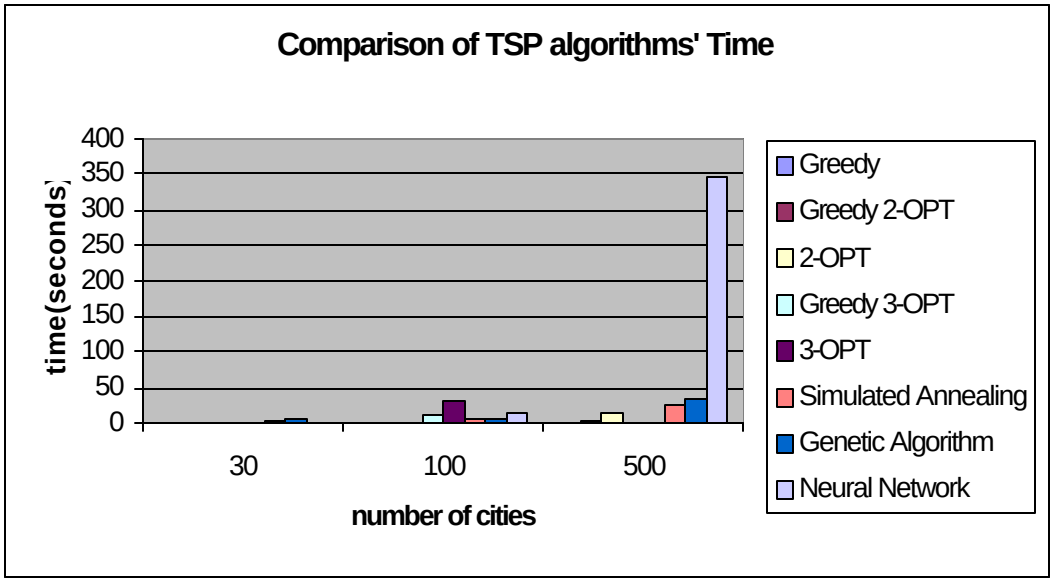
< Time Comparison >

Algorithm \ City Size	**30**	**100**	**500**
Greedy	0	0	1
Greedy 2-OPT	0	0	2
2-OPT	0	0	14
Greedy 3-OPT	0	11	
3-OPT	0	30	
Simulated Annealing	2	6	27
Genetic Algorithm	6	8	34
Neural Network	1	12	346

< Length Comparison >



< Time Comparison >



5. Conclusion

We implemented the above-mentioned heuristic algorithms. It is shown that the algorithms were improved by using non crossing methods. This improved methods always result in better solutions. The conclusion that is derived from this project is for small-size TSP ($n \leq 50$), greedy 2-opt algorithm performs pretty well, i.e high optimality vs. little computational time. The neural network(Self Organizing feature Maps) algorithm demonstrates the best efficiency for all city sizes. Through the simulation, the improved neural network algorithm gives us the shortest distance for 30 cities, 100 cities and 500cities. The number of the cities doesn't affect the optimality of algorithms. Therefore no matter how many cities are involved in the Travelling salesman problem, the Neural network algorithm will give the best solution of all these 16 algorithms that were mentioned in this report. The simulation of 3-Opt algorithm take a very long time for a large number of cities such as 500. It is not efficient to use 3-Opt algorithm for problem with more than 100 cites.

For small-size TSP ($n \leq 50$), improved greedy 2-opt algorithm is recommended. For medium-size TSP ($50 \leq n \leq 100$), improved 2-opt algorithm and neural network are recommended for their optimality and efficiency. For large-size problem ($100 \leq n \leq 500$), the improved genetic algorithm is recommended. For any problem-size, if the computational time is not a constraint, the improved neural network is always recommended.

6. References

Angeniol, B., Vaubois, G de L C. and Texier JYL.(1988) *Self-Organizing Feature Maps and the Traveling Salesman Problem*, Neural Networks, Vol 1, pp 289-293.

Heragu, Sunderesh (1997) *Facilities Design*, PWS Publishing Company, Boston, MA.