Automaty i gramatyki

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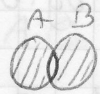
Zad. 1.

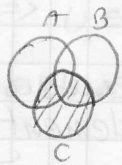
a) $(A \cup B) \setminus (A \cap B) \;|\; (A \setminus B) \cup (B \setminus A)$

to samo

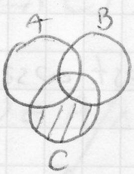


b) $(C \setminus A) \cap (C \setminus B) \;|\; C \setminus (A \cap B)$

nie to samo

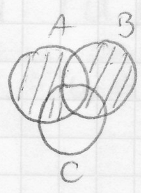


c) $(C \setminus A) \cap (C \setminus B) \;|\; C \setminus (A \cup B)$



to samo

d) $(A \setminus B) \cup (B \setminus A) \;|\; (A \cup B) \setminus (A \cap B)$



to samo

Zad. 2

Ciągi i podciągi

* 5 8 4 4 5 0 0

a) ile jest prefiksów właściwych?

6 (5, 58, 584, ...)

b) ile jest podciągów 2-elementowych, żeby nie było powtórzeń?

~~5~~ + ~~6~~ 11 → 58, 54, 55, 50, 84, 85, 80, 44, 45, 40, ~~50~~, 60

c) ile jest 2-elementowych podciągów spójnych? (elementy są obok siebie)

6

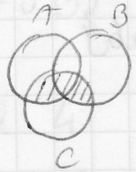
d) zbiór n-elementowy: ile jest wszystkich podzbiorów tego zbioru?

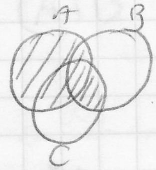
2^n

Zad. 3

Zbiory A, B, C są językami

a) $(A \cup B) \cap C \neq A \cup (B \cap C)$





b) $A(B \cup C) = AB \cup AC$

$AB = \{uv : u \in A \land v \in B\}$

$uv : u \in A \land v \in (B \cup C) \Leftrightarrow u \in A \land (v \in B \lor v \in C)$

$\Leftrightarrow (u \in A \land v \in B) \lor (u \in A \land v \in C)$

$\Downarrow$

$AB \cup AC$

c) $A(B \cap C) \neq AB \cap AC$

$uw : u \in A \land (w \in B \land w \in C) \Leftrightarrow$
$u \in A \land w \in B \land w \in C$ } źle

$uw : (u \notin A \land w \in B) \land (u \notin A \land w \in C) \Leftrightarrow$

$u \in A \land w \in B \land w \in C$

np. $A = \{a, \varepsilon\}$
$B = \{b\}$ $\rightsquigarrow$ $B \cap C = \{\varepsilon\}$
$C = \{a, b\}$

$AB = \{ab, b\}$
$AC = \{aab, ab\}$
$AB \cap AC = \{ab\}$

d) $(AB)^* \neq A^* B^*$

def: $A^* = \{\varepsilon\} \cup A \cup AA \cup AAA \cup \ldots$

$(AB)^* = \{ab, ab, ab\}$
$A^* = \{a, aa, aaa\}$ $A^* B^* = \{ab, aabb, \ldots\}$
$B^* = \{b, bb, bbb\}$

e) $(A \cup B)^* \neq A^* B^*$

f) $(A \cup B)^* = (A^* B^*)^*$
np. $A = \{a, b\} \rightsquigarrow A^* = \{aab, baa, \ldots\}$
trzeba udowodnić zawieranie się zbiorów
$(A \cup B)^* \supseteq (A^* B^*)^*$
$\left.\begin{matrix} A^* B^* \supseteq A \\ A^* B^* \supseteq B \end{matrix}\right\} A^* B^* \supseteq A \cup B = (A^* B^*)^* \supseteq (A \cup B)^*$

z def: $A \subseteq B = A^* \subseteq B^*$

g) $(A \cap A^*)^* = A^*$

h) $A^* \setminus A^* = \{\varepsilon\}$

i) $(A \cup B)^* \setminus (A^* \cup B^*) = (A \setminus B \cup B \setminus A)^*$

Zad. 4

Alfabety

a) $(\Sigma\Sigma)^*$ = wszystkie słowa dł. parzystej

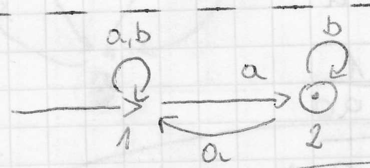
b) $\{aa, ab, ba, bb\}^*$ = słowa parzystej dł. z liter "a" i "b"

c) $\{a, ab, ba, baba\} \cap \{ab, ba\}^2 = \{baba\}$

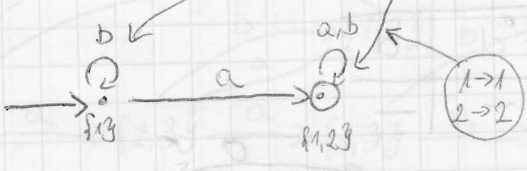
$\Downarrow$

$\{abba, baba, abab, baba, baab\}$

Np.



	a	b
1	1,2	1
2	1	2



1 → 1, 2

2 → 1

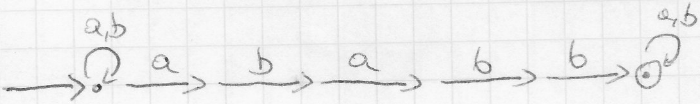
suma zbiorów

Stany akceptujące tam, gdzie stan zawiera liczbę z pierwotnego automatu (tu "2")

Zad. 1.

Narysuj automat niedeterministyczny, który akceptuje słowa zawierające podsłowo

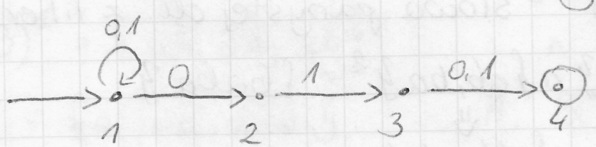
a) ababb

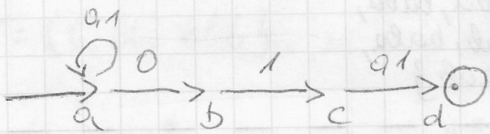


Zad. 2

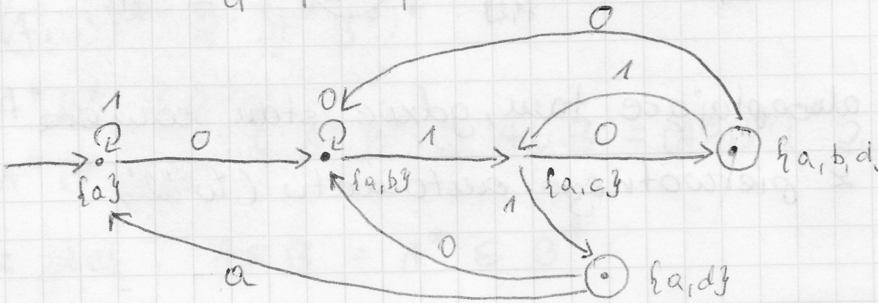
Zdeterminizować automaty

a)

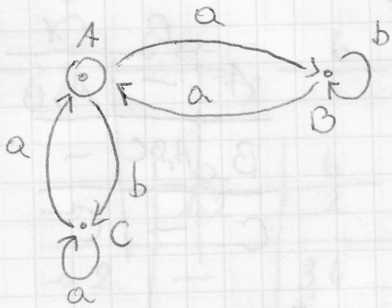




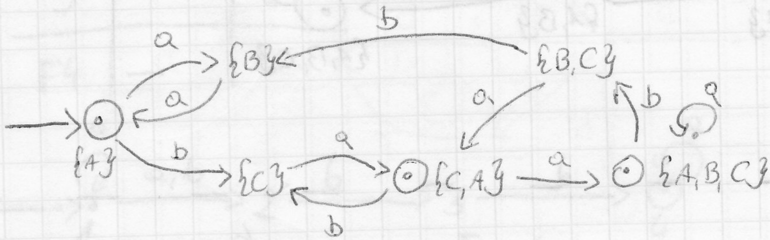
	0	1
a	a,b	a
b	—	c
c	d	d
d	—	—



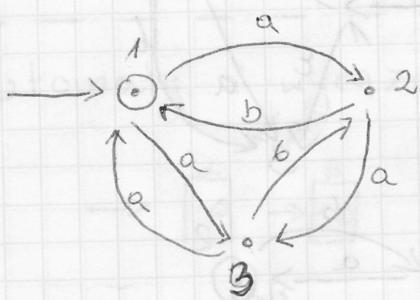
b)



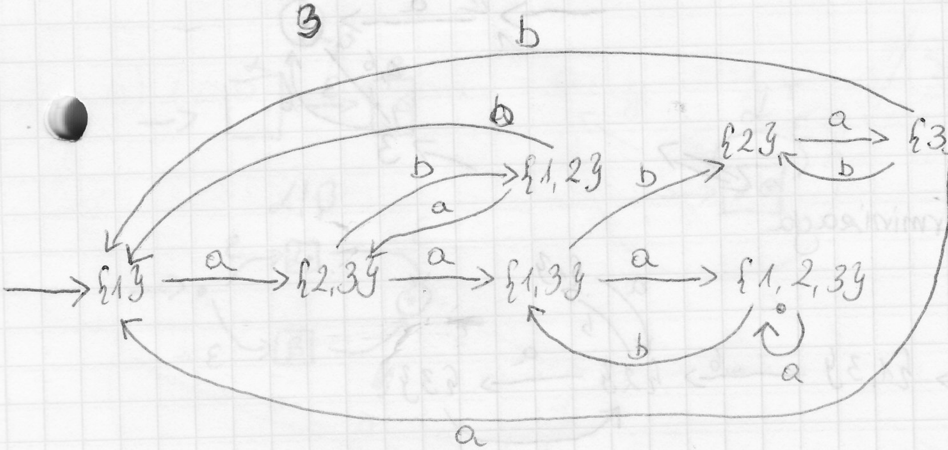
	a	b
A	B	C
B	A	B
C	C, A	–



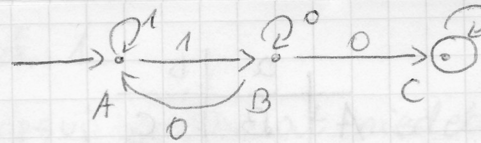
c)



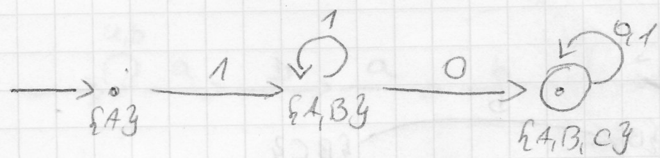
	a	b
→ F1	2, 3	–
2	3	1
3	1	2



d)



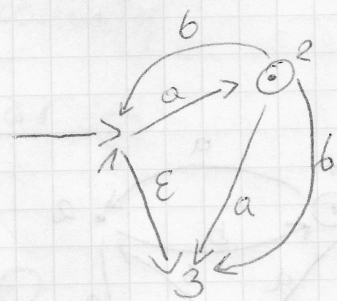
	0	1
A	—	A,B
B	A,B,C	—
C	—	C

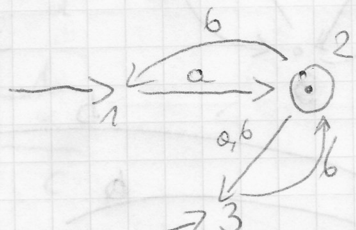


ε-przejścia

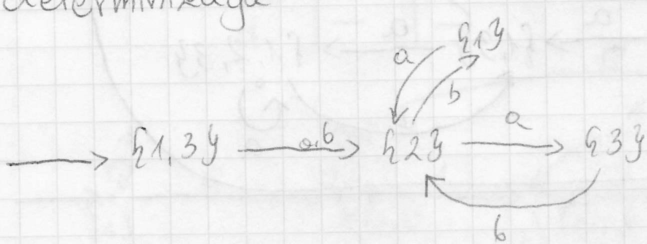
stany akcept.

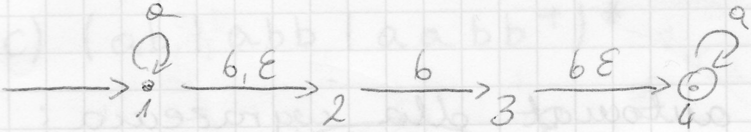
	a	b
→1	2	—
F2	3	1,3
→3	—	2



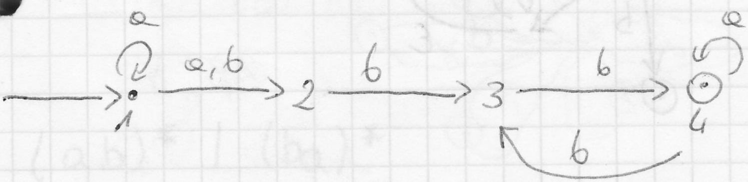


determinizacja



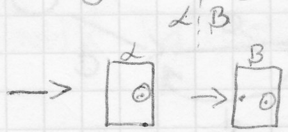


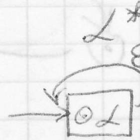
	a	b
→1	1,2	2
→2	—	3,4
3	—	4
F4	—	—

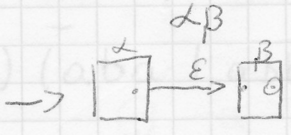


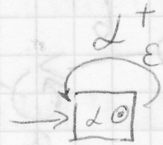
← ?

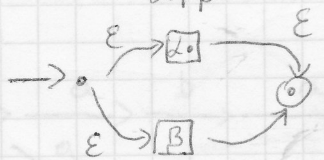
Automaty i wyrazenia regularne







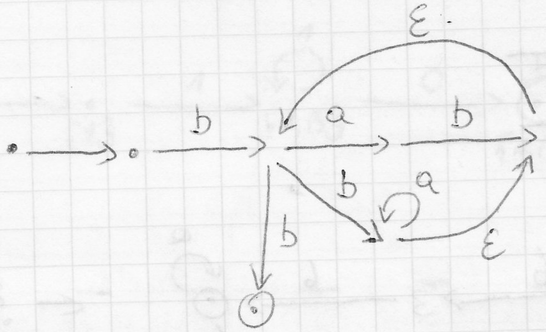




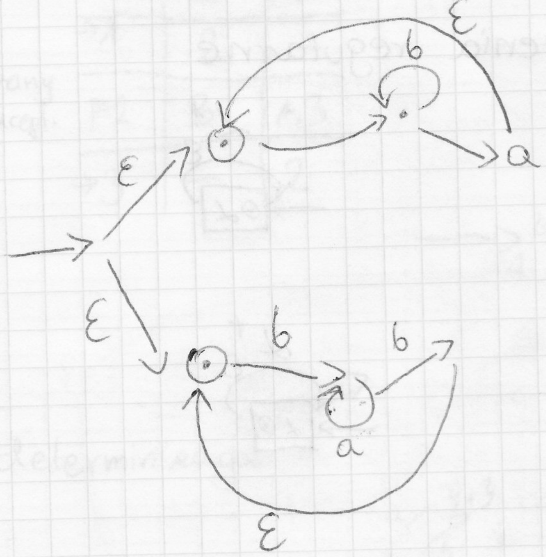
Zad. 1

Zbudować automat dla wyrażenia:

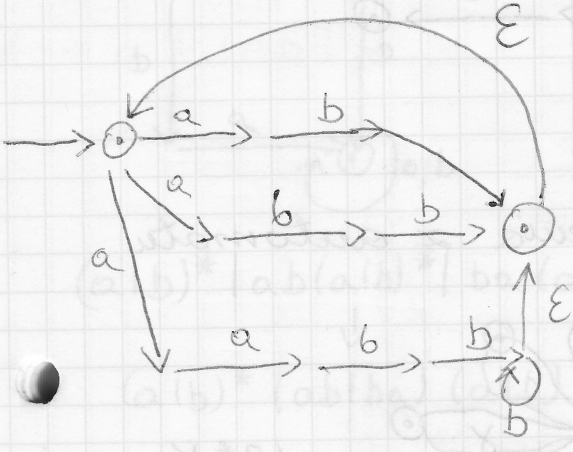
a) $b(ab \mid ba^*)^* b$



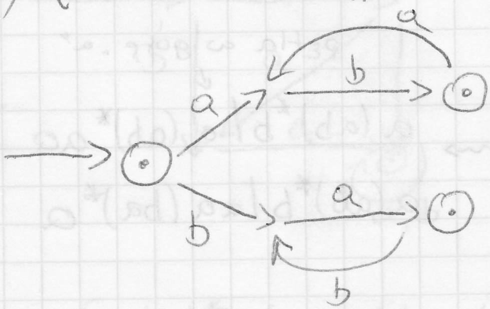
b) $(ab^*a)^* \mid (ba^*b)^*$



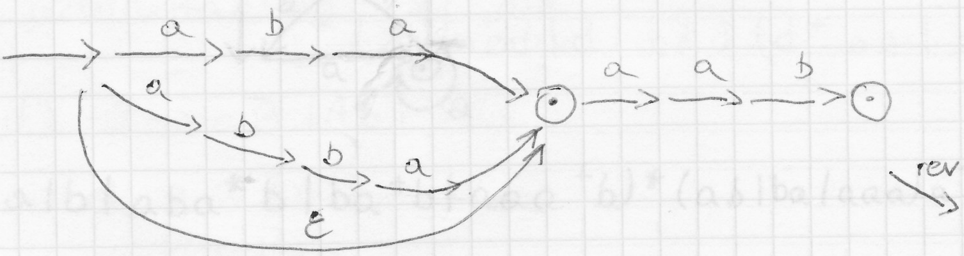
c) $(ab \mid abb \mid aabb^{+})^{*}$

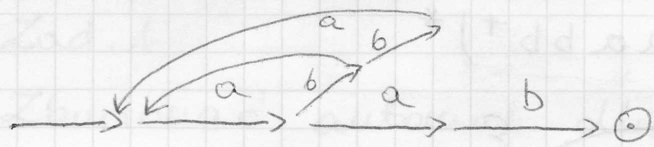


d) $(ab)^{*} \mid (ba)^{*}$



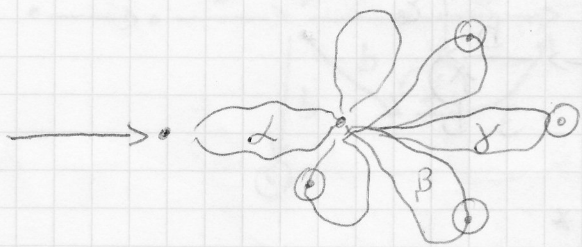
e) $(aba \mid abba)^{*}\, aab$





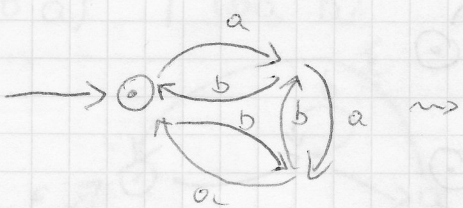
Zad. 2

Budowa wyrażenia z automatu



$\alpha\beta^*\gamma$

np.



pętla w górę „a"

$a(ab)^*b \mid a(ab)^*aa$

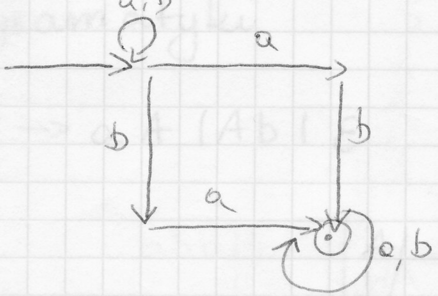
$a(ab)^*b \mid aa(ba)^*a$

pętla w górę "b"

$b(ba)^*a \mid b(ba)^*bb$

całość: $(a(ab)^*b \mid a(ab)^*aa \mid b(ba)^*a \mid b(ba)^*bb)^*$

a)

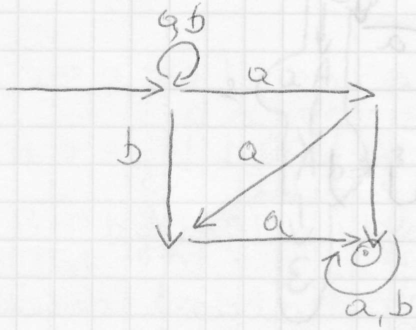


$$(a|b)^*|ab(a|b)^*|ba(a|b)^*$$

$$\downarrow$$

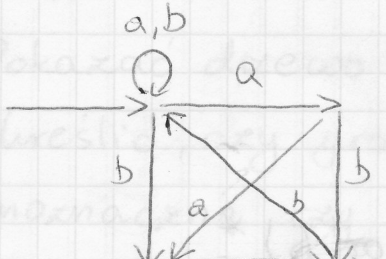
$$(a|b)^*(ab|ba)(a|b)^*$$

b)



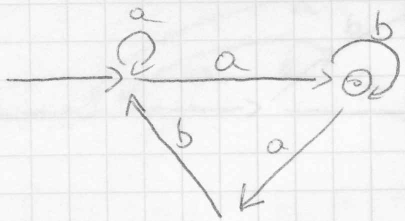
$$(a|b)^*(ab|ba|aaa)(a|b)^*$$

c)



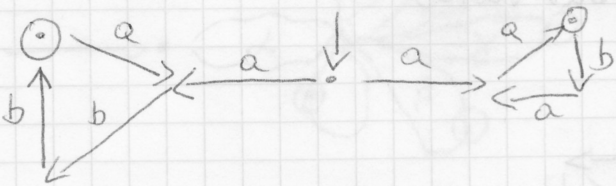
$$(a|b|aba^*b|ba^+b|aaa^+b)^*(ab|ba|aaa)a^*$$

d)



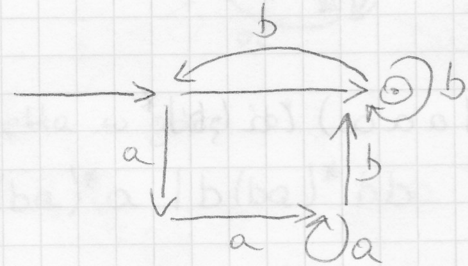
$(a^+ b^* ab)^* a^+ b^*$

e)



$a(bb(abb)^* \mid a(baa)^*)$

f)

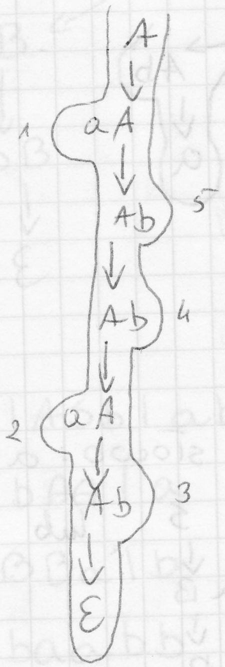


$(a \mid aa^+b)(ba \mid b \mid baa^+b)^*$

Gramatyki

$A \to aA \mid Ab \mid \varepsilon$

słowo: a a b b b
1 2 3 4 5



Zad. 1.

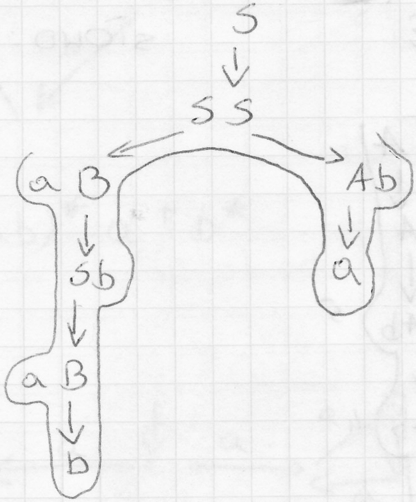
Pokazać drzewo wyprowadzenia i określić, czy gramatyka jest jednoznaczna czy nie.

a) $S \to aB \mid Ab \mid SS$

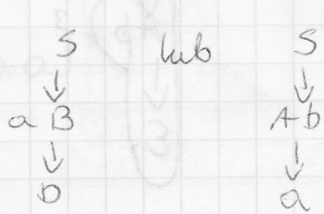
$A \to a \mid aS$

$B \to b \mid Sb$

• słowo: aabbab



nie jednoznaczna np. słowo: ab

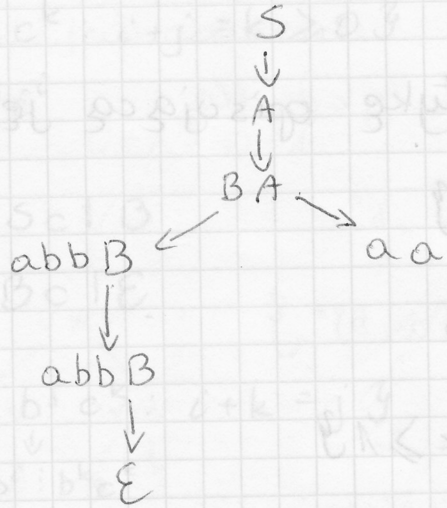


b) $S \to A \mid B \mid AB$

$A \to BA \mid aaA \mid abaA \mid aa$

$B \to abbB \mid \varepsilon$

słowo: abbabbaa

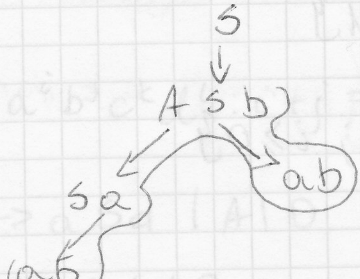


c) $S \to aSB \mid ASb \mid ab$

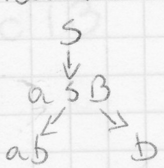
$A \to Sa \mid bAA \mid a$

$B \to bS \mid BBa \mid b$

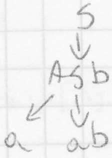
słowo: abaabb



gram. jednoznaczna: np. aabb



lub



Zad. 2

Napisać gramatykę opisującą język

a) $\{a^n b^{2n} : n \geq 1\}$

$S \to aSbb \mid abb$

b) $\{a^n b^k : n \geq 2k \geq 1\}$

$S \to aaASb \mid aaAb$

$A \to aA \mid \varepsilon$

c) $\{a^{2n} b^n : n \geq 1\}$

$S \to aaSb \mid aab$

b') $\{a^n b^k : n \geq 2k \geq 1\}$

$\downarrow$

$\{a^i a^{2k} b^k : k \geq 1;\ i \geq 0\}$

$S \to AS$

$A \to aA \mid \varepsilon$

d) $\{a^i b^j c^k : i+j=k \geq 0\}$

$a^i b^j c^j c^i$

$S \to aSc \mid B$

$B \to bBc \mid \varepsilon$

e) $\{a^i b^j c^k : i+k=j\}$

$a^i b^i : b^k c^k$

$S \to AB$

$A \to aAb \mid \varepsilon$

$B \to aBc \mid \varepsilon$

f) $\{a^i b^j c^k d^l : i+l=j+k\}$

?

g) $\{a^i b^j c^k d^l : i+j=k+l\}$

$S \to aSd \mid A \mid D$

$A \to bAd \mid B$

$B \to bBc \mid \varepsilon$

$D \to aDc \mid B$

Zad. 1

Zbudować gramatykę dla:

a) x - literki „a" i „b"

$\{x c\, rev(x) ; x \in \{a, b\}^*\}$

$S \to aSa \mid bSb \mid C$

ilość liter

b) $\{xcy : \#a(x) = \#a(y) ; x, y \in \{a, b\}^*\}$

$S \to aSa \mid bS \mid Sb \mid c$

c) $\{a^i b^j c^k : i + k = 2j\}$

$S \to AB \mid aAbBc$ ← nieparzyście

$A \to aaAb \mid \varepsilon$

$B \to bBcc \mid \varepsilon$

} generuje a, c parzyście

— — — —

d) 0, 1, +, *, (,) → elementy języka

zbudować gramatykę opisującą poprawne wyr. arytmetyczne

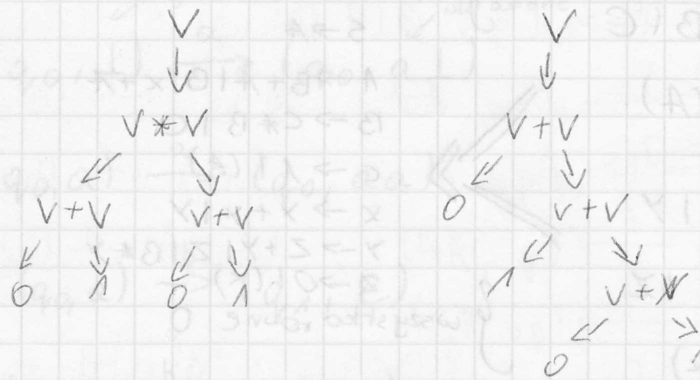
$C \to 0 \mid 1$

$V \to B \mid C \mid V+V \mid V*V$

$B \to (V)$

Czy jest jednoznaczna

$0+1*0+1$

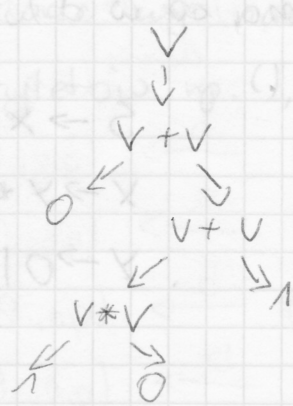


d') jednoznaczna

$S \to X + S \mid X$

$X \to Y * X \mid Y$

$Y \to 0 \mid 1 \mid (S)$



e) 0, 1, +, *, (,) → wrt. wyr. musi być min 1

A → B + A | B

B → C * B | C

C → 1 | (A)

} wszystko będzie dodatnie

X → Y + X | Y

Y → Z + Y | Z

Z → 0 | (X)

} wszystko równe 0

S → A

A → B + A | B | X + A

B → C * B | C

C → 1 | (A)

X → Y + X | Y

Y → Z + Y | Z | B * Y

Z → 0 | (X)

f) to samo, co w d') tylko z negacją

S → X + S | X

X → Y * X | Y

Y → 0 | 1 | (S) | -Y

Zad. 1

a) automat $a^n b^n$

odejmowane / wkładane

$(q_0, \perp) \xrightarrow{a} (q_0, a\perp)$

$(q_0, a) \xrightarrow{a} (q_0, aa)$

$(q_0, a) \xrightarrow{b} (q_1, \varepsilon)$

$(q_1, a) \xrightarrow{b} (q_1, \varepsilon)$

b) automat ma rozpoznawać wyr. nawiasowe z „(", „)", „[", „]" bez priorytetów np. (), ([]), [()], ~~()~~

$(q_0, \perp) \xrightarrow{(} (q_0, A\perp)$

$(q_0, \perp) \xrightarrow{[} (q_0, B\perp)$

$(q_0, A) \xrightarrow{)} (q_0, \varepsilon)$

$(q_0, B) \xrightarrow{]} (q_0, \varepsilon)$

$(q_0, A) \xrightarrow{(} (q_0, AA)$ $(q_0, B) \xrightarrow{(} (q_0, AB)$

$(q_0, A) \xrightarrow{[} (q_0, BA)$ $(q_0, B) \xrightarrow{[} (q_0, BB)$

c) wyrażenia nawiasowe z priorytetem

$(q_0, \perp) \xrightarrow{(} (q_0, A\perp)$

$(q_0, \perp) \xrightarrow{[} (q_0, B\perp)$

$(q_0, A) \xrightarrow{)} (q_0, \varepsilon)$

$(q_0, B) \xrightarrow{]} (q_0, \varepsilon$

$(q_0, A) \xrightarrow{(} (q_0, AA)$

$(q_0, B) \xrightarrow{(} (q_0, AB)$

$(q_0, B) \xrightarrow{[} (q_0, BB)$

d) $\{w \in (a,b)^* : \#a(w) = \#b(w)\}$

$(q_0, \perp) \xrightarrow{a} (q_0, A\perp)$

$(q_0, \perp) \xrightarrow{b} (q_0, B\perp)$

$(q_0, A) \xrightarrow{a} (q_0, AA)$

$(q_0, B) \xrightarrow{b} (q_0, BB)$

$(q_0, A) \xrightarrow{b} (q_0, \varepsilon)$

$(q_0, B) \xrightarrow{a} (q_0, \varepsilon)$

e) $\{w \in (a^i b^{2i}),\ i \geq 0\}$

$(q_0, \perp) \xrightarrow{a} (q_0, A\perp)$ $\quad$ $(q_0, \perp) \xrightarrow{a} (q_0, AA\perp)$

$(q_0, A) \xrightarrow{a} (q_0, AA)$ $\quad$ $(q_0, A) \xrightarrow{a} (q_0, AAA)$

$(q_0, A) \xrightarrow{b} (q_1, B)$ $\quad$ lub $\quad$ $(q_0, A) \xrightarrow{b} (q_1, \varepsilon)$

$(q_1, B) \xrightarrow{b} (q_1, \varepsilon)$ $\quad$ $(q_1, A) \xrightarrow{b} (q_1, \varepsilon)$

$(q_1, A) \xrightarrow{b} (q_1, B)$

f) $\{a^i b^j a^j b^i\ ;\ i, j > 0\}$

$(q_0, \perp) \xrightarrow{a} (q_0, A\perp)$

$(q_0, A) \xrightarrow{a} (q_0, AA)$

$(q_0, A) \xrightarrow{b} (q_0, BA)$

$(q_0, B) \xrightarrow{b} (q_0, BB)$

$(q_0, B) \xrightarrow{a} (q_1, \varepsilon)$

$(q_1, A) \xrightarrow{b} (q_1, \varepsilon)$

$(q_1, B) \xrightarrow{a} (q_1, \varepsilon)$

g) palindrom $(a,b) \rightsquigarrow \{ w \in (a,b)^* : w = rev(w) \}$

$(q_0, \perp) \xrightarrow{a} (q_0, A\perp)$

$(q_0, \perp) \xrightarrow{b} (q_0, B\perp)$

$(q_0, A) \xrightarrow{a} (q_0, AA)$

$(q_0, B) \xrightarrow{b} (q_0, BB)$

$(q_0, A) \xrightarrow{b} (q_0, BA)$

$(q_0, B) \xrightarrow{a} (q_0, AB)$

$(q_0, A) \xrightarrow{a} (q_1, \varepsilon)$

$(q_0, B) \xrightarrow{b} (q_1, \varepsilon)$

$(q_0, A) \xrightarrow{a,b} (q_1, A)$

$(q_0, B) \xrightarrow{a,b} (q_1, B)$

$(q_1, A) \xrightarrow{a} (q_1, \varepsilon)$

$(q_1, B) \xrightarrow{b} (q_1, \varepsilon)$