CAŁKI

$$\int \frac{f'}{f} = \ln|f| + c$$

c – stała ; c’=0

$$\int \frac{2x}{x^2+1} \quad ; \quad (x^2+1)' = 2x$$

Przykład:

a)

$$\int tg^2x\,dx = \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx = \int \frac{1}{\cos^2 x}\,dx - \int 1dx = tgx - x + c$$

b)

$$\int \frac{e^{-2x}-4}{e^{-x}+2}\,dx = \int \frac{\cancel{(e^{-x}+2)}(e^{-x}-2)}{\cancel{e^{-x}+2}}dx = \int e^{-x}\,dx - \int 2\,dx = -e^{-x} - 2x + c$$

Całkowanie przez części

$$\int u'v = uv - \int u * v'$$

Przykład:

a) zadanie z kolokwium

$$\int x * arctgx\,dx = \left\| \begin{matrix} u' = x \;;\; u = \frac{1}{2}x^2 \\ v = arctgx \;;\; v' = \frac{1}{1+x^2} \end{matrix} \right\| = \frac{1}{2}x^2 * arctgx - \int \frac{1}{2}x^2 * \frac{1}{1+x^2}\,dx =$$

$$= \frac{1}{2}x^2 * arctgx - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}\,dx = \frac{1}{2}x^2 * arctgx - \frac{1}{2}\int \frac{x^2+1}{x^2+1}\,dx - \frac{1}{2}\int \frac{-1}{x^2+1}\,dx =$$

$$= \frac{1}{2}x^2arctgx - \frac{1}{2}x + \frac{1}{2}arctgx + c$$

b)

$$\int e^{2x} * \sin x\;dx = \left\| \begin{matrix} u; = e^{2x} \;;\; u = \frac{1}{2}e^{2x} \\ v = \sin x \;;\; v' = \cos x \end{matrix} \right\| = \frac{1}{2}e^{2x} * \sin x - \int \frac{1}{2}e^{2x} * \cos x\;dx =$$

$$= \frac{1}{2}e^{2x} * \sin x - \frac{1}{2}\int e^{2x} * \cos x\;dx = \left\| \begin{matrix} u' = e^{2x} \;;\; u = \frac{1}{2}e^{2x} \\ v = \cos x \;;\; v' = -\sin x \end{matrix} \right\| = \frac{1}{2}e^{2x} * \sin x -$$

$$-\frac{1}{2}\left(\frac{1}{2}e^{2x} * \cos x + \int \frac{1}{2}e^{2x} * \sin x\;dx\right)$$

$$\int e^{2x} * \sin x\;dx = \frac{1}{2}e^{2x} * \sin x - \frac{1}{4}e^{2x} * \cos x - \frac{1}{4}\int e^{2x} * \sin x\;dx$$

$$\frac{5}{4}\int e^{2x} * \sin x\;dx = \frac{1}{2}e^{2x} * \sin x - \frac{1}{4}e^{2x} * \cos x$$

$$\int e^{2x} * \sin x\;dx = \frac{4}{10}e^{2x} * \sin x - \frac{1}{5}e^{2x} * \cos x + c$$

c)

$$\int \sin^2 x \; dx = \int \sin x * \sin x \; dx = \left\| \begin{matrix} u' = \sin x & ; & u = -\cos x \\ v = \sin x & ; & v' = \cos x \end{matrix} \right\|$$

$$\int \sin^2 x \; dx = -\cos x * \sin x - \int (-\cos x) * \cos x \; dx = -\cos x * \sin x - \int -\cos^2 x \; dx =$$

$$= -\cos x * \sin x + \int \cos^2 x \; dx = -\cos x * \sin x + \int (1 - \sin^2 x) \, dx =$$

$$= -\cos x * \sin x + \int 1 \, dx - \int \sin^2 x \; dx = \int \sin^2 x \; dx \quad \because \text{równanie} \because$$

$$\int \sin^2 x \; dx = -\cos x * \sin x + x - \int \sin^2 x \; dx$$

$$2 \int \sin^2 x \; dx = -\cos x * \sin x + x$$

$$\int \sin^2 x \; dx = \frac{-\cos x * \sin x + x}{2} + c$$

$$\cos^2 x = 1 - \sin^2 x$$

d)

$$\int (5 - 3x)^{10} \, dx$$

$$T = 5 - 3x$$

$$1 * dT = -3dx \; \because / -3$$

$$dx = \frac{dT}{-3}$$

$$\int T^{10} * \frac{dT}{-3} = -\frac{1}{3} * \frac{1}{11} * T^{11} + c = -\frac{1}{3} * \frac{1}{11} * (5 - 3x)^{11}$$

$$\int f + g = \int f + \int g$$

$$\int x^n \; dx = \frac{1}{n+1} * x^{n+1} + c$$

e)

$$\int x^2 * \sqrt[5]{5x^3 + 1} \, dx = \int \sqrt[5]{5x^3 + 1} * x^2 \, dx$$

$$T = 5x^3 + 1$$

$$1 * dT = (5x^3)' \quad ; \quad (5x^3)' = 3x^2 * (5x)' = 3x^2 * 5$$

$$dT = 15x^2 \, dx$$

$$x^2 dx = \frac{dT}{15}$$

$$\int \sqrt[5]{5x^3 + 1} * x^2 \, dx = \int \sqrt[5]{T} * \frac{dT}{15} = \frac{1}{15} \int T^{\frac{1}{5}} \, dT = \frac{1}{15} * \frac{1}{\frac{1}{5} + 1} * T^{\frac{1}{5}+1} + c =$$

$$= \frac{1}{15} * \frac{1}{\frac{1}{5} + 1} * (5x^3 + 1)^{\frac{1}{5}+1} + c$$

f)

$$\int \frac{\sqrt{1+\ln x}}{x}\,dx = \int \sqrt{1+\ln x} * \frac{1}{x}\,dx = \int \sqrt{T}\,dT = \int T^{\frac{1}{2}}\,dT = \frac{1}{\frac{1}{2}+1} * T^{\frac{1}{2}+1} + c =$$

$$= \frac{1}{\frac{1}{2}+1} * (1+\ln x)^{\frac{1}{2}+1} + c$$

$$T = 1 + \ln x$$

$$dT = \frac{1}{x}\,dx$$

$$\int f = F \text{ to } F' = f$$

g) z egzaminu

$$\int \frac{-6x^2+1}{\sqrt{3x-6x^3}}\,dx$$

$$(3x-6x^3)' = 3 - 18x^2 = 3(1-6x^2)$$

$$T = 3x - 6x^3$$

$$dT = 3(1-6x^2)dx \implies dx = \frac{dT}{3(1-6x^2)}$$

$$\int \frac{-6x^2+1}{\sqrt{3x-6x^3}}\,dx = \int \frac{\cancel{(-6x^2+1)}}{\sqrt{T}} * \frac{dT}{3\cancel{(1-6x^2)}} = \int \frac{dT}{3\sqrt{T}} = \frac{1}{3}\int \frac{dT}{T^{\frac{1}{2}}} = \frac{1}{3}\int dT * T^{-\frac{1}{2}} =$$

$$= \frac{1}{3}\left(\frac{1}{-\frac{1}{2}+1}\right) * T^{-\frac{1}{2}+1} + c = \frac{1}{3}\left(\frac{1}{-\frac{1}{2}+1}\right) * (3x-6x^3)^{-\frac{1}{2}+1} + c$$

$$\int \frac{-6x^2+1}{\sqrt{3x-6x^3}}\,dx =$$

h) z egzaminu

$$\int x * e^{x^2} * (x^2+1)\,dx$$

i) z egzaminu

$$\int \frac{x^2}{\cos^2(x^3+1)}\,dx$$

j) z egzaminu

$$\int \frac{\sqrt{x}}{1+x}\,dx$$

k) z egzaminu

$$\int \frac{dx}{\sqrt{1-4x^2}}$$