CIS 525 Software Development of Parallel and Distributed Systems

C/E PETRI-NETS – SOME DEFINITIONS AND EXAMPLES

1. **Subnet:**

Let $N_1 = (B_1, E_1, F_1)$ and $N_2 = (B_2, E_2, F_2)$ be a pair of nets. Then N_1 is a subnet of N_2 if and only if $B_1 \subseteq B_2$ and $E_1 \subseteq E_2$ and $F_1 = F_2 \cap ((B_1 \times E_1) \cup (B_2 \times E_2))$

Example 1: N_1 is a subset of N_2:

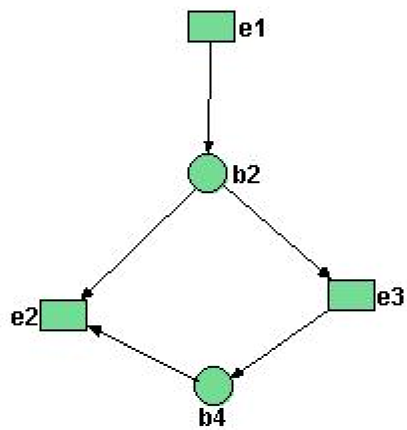

Figure 1: Subnet N_1

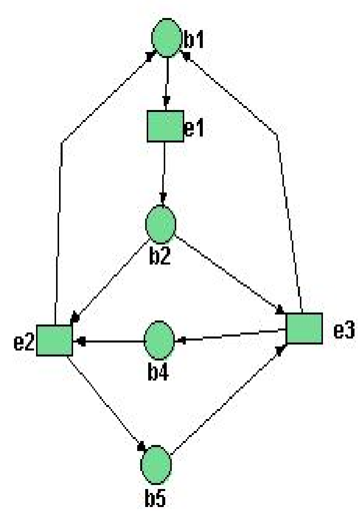

Figure 2: Net N_2

2. Dual Net:

Let N = (B,E,F) be a net. Then the dual of N, denoted as $\hat{N}$, is the triplet $\hat{N} = (\hat{B}, \hat{E}, \hat{F})$, where $\hat{B}$ = E, $\hat{E}$ = B and $\hat{F} = F^{-1}$

Example 2: The dual of N_2 from example 1 is:

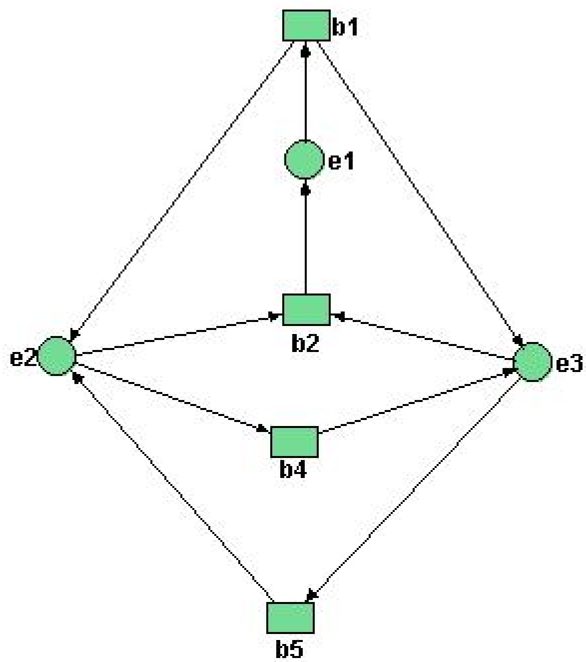

Figure 3: Dual net $\hat{N}_2$

Theorem 1: Let N_1 and N_2 be two nets

i> $\hat{N}_1$, the dual of N_1 is also a net.

ii> $\hat{\hat{N}}_1 = N_1$

iii> N_1 is a subnet of N_2 ⇔ $\hat{N}_1$ is a subnet of $\hat{N}_2$

Why one discusses contact-free C/E systems?

1> the simplest possible C/E systems
2> the notion of non-sequential process generated by C/E systems can be formulated in a clean way only for contact-free C/E systems
3> contact-free C/E nets can be generated smoothly into arbitrary C/E net.

Definition: The C/E system $N=(B,E,F,C_{in})$ is contact-free if and only if

$$\forall_{e \in E} \; \forall_{C \in C_N} \; [\,{}^{\bullet}e \subseteq C \Rightarrow e^{\bullet} \cap C = \emptyset\,]$$

Theorem 2: Let $N=(B,E,F,C_{in})$ is contact-free C/E system, $C \in C_N$ and $G \subseteq E$. Then

$$C[G> \text{ iff } {}^{\bullet}G \subseteq C \text{ and } \forall_{e_1,e_2 \in G} [e_1 \neq e_2 \Rightarrow {}^{\bullet}e_1 \cap {}^{\bullet}e_2$$

Fundamental behavioral situations:

For given case C of a C/E system, two events e_1, e_2 can be related to each other in at least 3 ways:

a> Sequence: e_1 can occur at C but not e_2; however, after e_1 has occurred e_2 can occur.

b> Conflict: e_1 and e_2 can occur individually at C but not both; in other words $\{e_1\}$ and $\{e_2\}$ are steps at C while $\{e_1,e_2\}$ is not a step at C.

c> Concurrency: both e_1 and e_2 can occur at C with no order specified over their occurrences. In other words, $\{e_1,e_2\}$ is a step at C.

Fact: Net theory separates these relationships conceptually, graphically, and mathematically.

<u>Sequence:</u>

<u>Graphically</u>

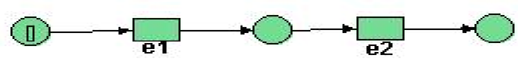

Figure 4. A sequence of two events e1 and e2.

Conceptually: "Occurrence of e_2 must be preceded by that of e_1"

Definition: Let $C \in C_N$, and $e_1, e_2 \in E_N$, where N is a C/E system; we say, mathematically: e_1 and e_2 are in sequence at C $\Leftrightarrow$ $C[e_1>$ and $\neg\,(C[e_2>)$ and $C'[e_2>$ where $C[e_1>C'$

<u>Conflict</u>: e_1 and e_2 can occur individually; but due to "shared" condition b, $\{e_1, e_2\}$ is not a step.

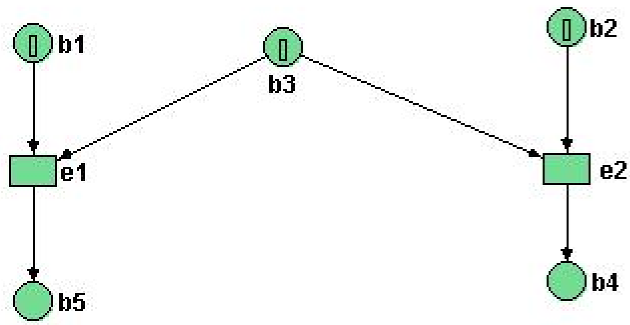

Figure 5. Petri net with conflict of events.

Reachability graph

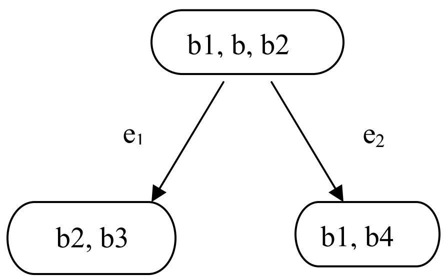

Figure 6. Fragment of Reachability graph.

Fact: either e_1 or e_2 can occur ≡ non-determinism

Definition: Let e_1 and e_2 be two events and C a case of a C/E system. e_1 and e_2 are in conflict at C iff $C[e_1>$ and $C[e_2>$ but not $C[\{e_1,e_2\}>$

Concurrency:

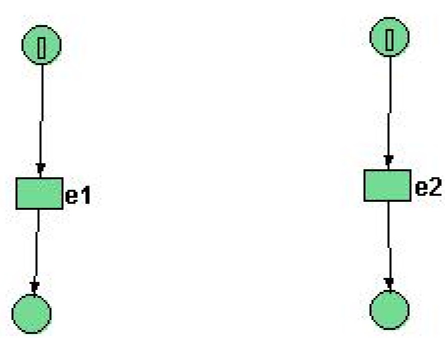

Figure 7. Two concurrent events.

e_1 and e_2 can occur without interfering with each other. No order is specified over their occurrences. Hence in general the occurrences of events and the resulting holdings of conditions will be partially ordered; C/E system can exhibit non-sequential behavior.

Definition: Let e_1 and e_2 be two events and C a case of the C/E system. e_1 and e_2 can occur concurrently at C iff $C[\{e_1, e_2\}>$

Conclusions:

1> Sequence= sequential behavior = linear ordering of events
2> Conflict= non determinism(with restrictions)= choice
3> Concurrency= non-sequentially behavior= partial ordering of events

Confusion = concurrency + conflict

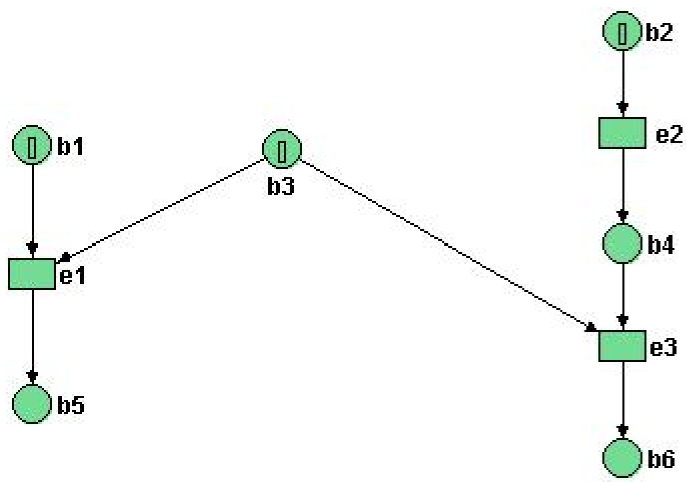

Figure 8. Petri net with confusion.

$C = \{b_1, b_2, b_3\}$
$C' = \{b_4, b_5\}$

$C[\{e_1,e_2\}>C'$

Ci confusion (Conflict – increasing confusion)

$Cfl\{e_1, C\} = \varnothing$
(C, e_1, e_2) is confusion because
$cfl(e_1,C) = \varnothing$, and $cfl(e_1,C_2) = e_3$, where $C_2 = \{b_1,b_2,b_3\}$

Disagreement over whether or not a conflict was resolved in going from the case C to C' via the step $\{e_1, e_2\}$. Potential interpretations:

Observer1 (no conflict point of view): e_1 occurred first without being in conflict with any other event. And then e_2 occurred.

Observer 2 (conflict point of view): e_2 occurred first. As a result e_1 and e_3 got in conflict. This conflict was resolved in favor of e_1 which then occurred.

Example: switching circuit confusion= glitch problem= synchronization failure problem

Fact:

1> Systems with confusion are difficult to analyze, because "the intermediate cases" determined by the elements of the step could differ. As a result one cannot take advantage of concurrency and analyze the cases generated just by one possible sequentialization of a step one must analyze every possible sequentialization.

2> Net theory suggests that it is not the combination of choice and concurrency that causes difficulties; rather it is those combinations of "choice" and "concurrency" resulting in confusion that cause trouble. Choice and concurrency can be combined in confusion - free manner.

3> It is not always possible to avoid confusion.

Example: (mutual exclusion problem)

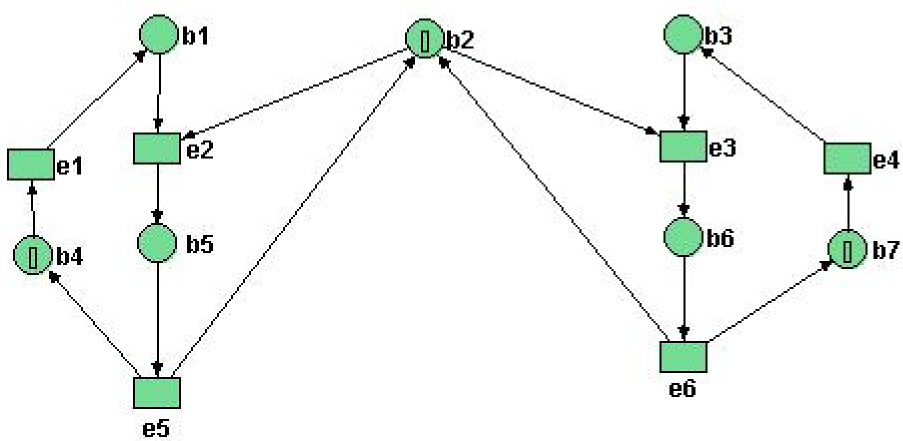

Figure 9. Petri net model of mutual exclusion.

Reachability Graph:

C={b2, b4, b7}

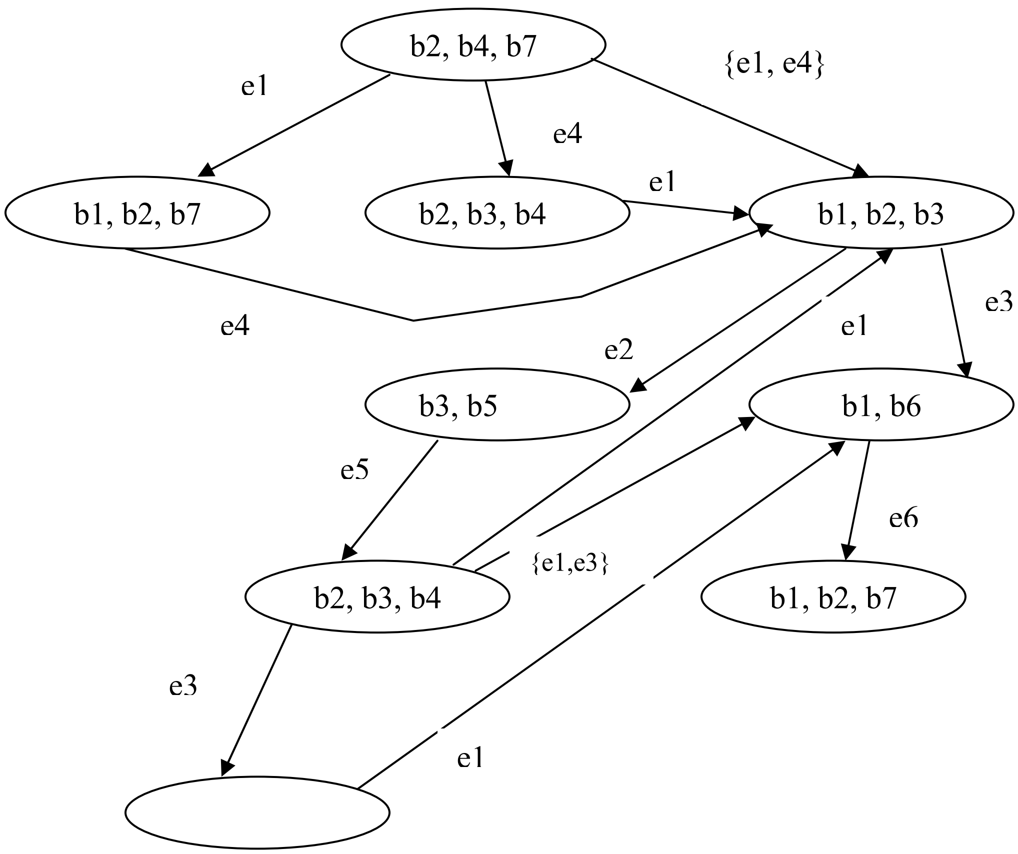

Figure 10. Reachability graph.

Formalization of confusion:

Definition: Let N= (B, E, F, C_N) be and C/E system, let C ϵ C_N and let e ϵ E be such that C[e>. The conflict set of e at C, denoted cfl(e, C) is defined

$$Cfl(e,c) = \{e' \epsilon \; E: C[e'> \text{and} \neg C[\{e,e'\}>\}$$

i.e. the conflict set of e at C is the set of all events that are in conflict with e at C.

Definition:
Let N= (B, E, F, C_{in}) be an C/E system let C ϵ C_N and let e_1, e_2 be two distinct events in E such that C[{e_1,e_2}>. The triplet(C, e_1, e_2) is a confusion at C. We say that N is confused at C iff, there is a confusion at C.

Thus a triplet (C,e_1,e_2) is a confusion if{e_1,e_2} is a step at C and the occurrence of e_2 at C change the conflict set of e_1.

Classification of confusion:

Let N be an C/E system, C ϵ C_N, e_1,e_2 ϵ E_N. Let γ = (C,e_1,e_2) be a confusion and let C[e_1>C_2.

(i) γ is a conflict – increasing confusion (c_i confusion)
iff cfl(e_1,C) $\subset$ cfl (e_1,C_2).

(ii) γ is a conflict – decreasing confusion (c_d confusion)
iff cfl(e1,C_2) $\subset$ cfl (e_1,C).

Example:

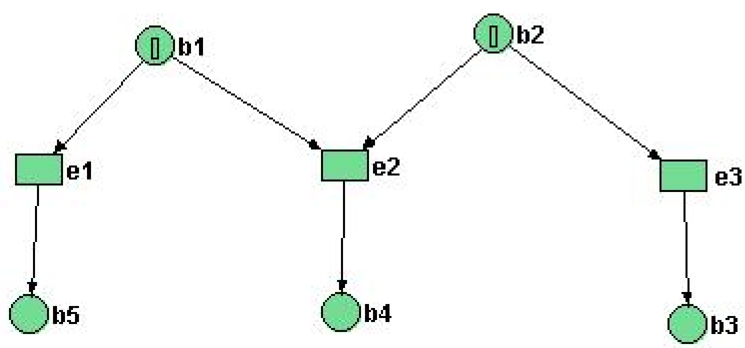

Figure 11. Illustration of confusion.

C = {b_1, b_2}

(C,e_1,e_2) is a confusion because, where C_2 = {b_1,b_3}

Since cfl (e_1,C_2) $\subset$ cfl (e_1,C)

(C, e_1, e_2) is a c_d confusion.

Example: Confusion that is neither c_i nor c_d confusion.

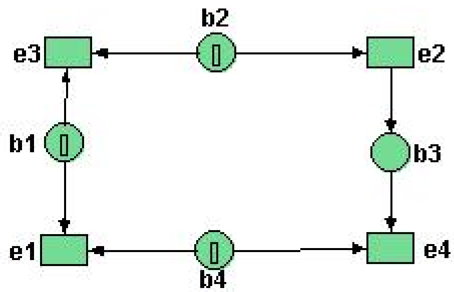

Figure 12. Illustration of confusion.

For C = {b_1,b_2,b_4}; (C,e_1,e_2) is a confusion because cfl (e_1,C) = {e_3} $\neq$ {e_4} = cfl(e_1,C_2) where C_2 = {b_1,b_3,b_4}. Note that (C,e_1,e_2) is neither a c_i confusion nor a c_d confusion.

Conclusions:

1. The distinctions between c_i and c_d confusion is not "exhaustive" there are confusions that are neither c_i nor c_d.

2. The fact that (C,e_1,e_2) is a confusion expresses certain "influence" of e_2 on e_1 at C. It is also important to know whether or not also e_1 can influence e_2 in a similar fashion.

Definition: Let γ = (C, e_1,e_2) be a confusion;

γ is symmetric ⇔ (C,e_2,e_1) is also a confusion other wise γ is asymmetric.

Examples:

1. γ = (C,e_1,e_2) is a c_i confusion that is asymmetric
(C,e_2,e_1) is not a confusion.

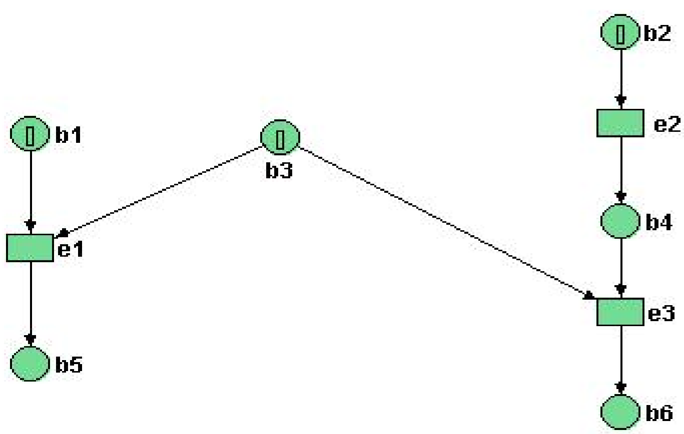

Figure 13. Illustration of confusion.

2.

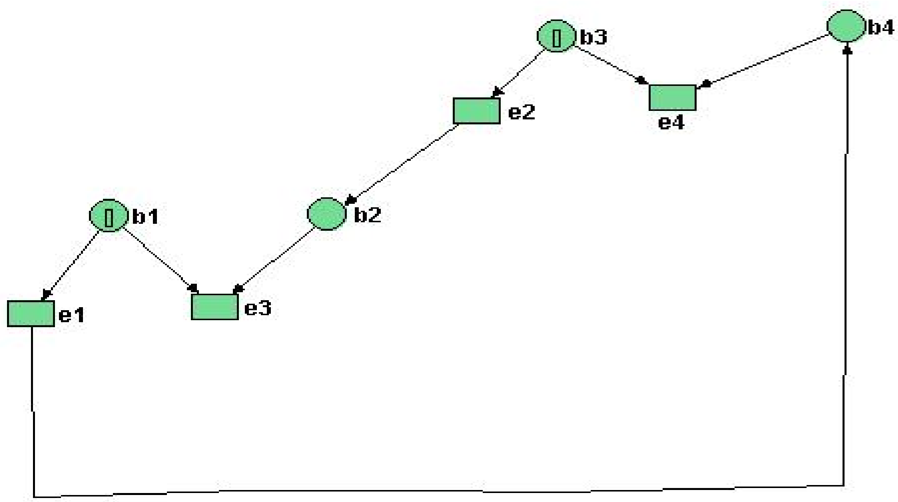

Figure 14. Illustration of confusion.

$C=\{b_1, b_3\}$
$\gamma = (C, e_1, e_2)$ is a c_i confusion. -->

$cfl(e,C) = \varnothing$
$cfl(e,C_2) = \{e_3\}$
where $C_2 = \{b_1,b_2\}$

$\gamma' = (C, e_2, e_1)$ is a c_i confusion -->

(C, e_1, e_2) is a c_i confusion that is symmetric

3.

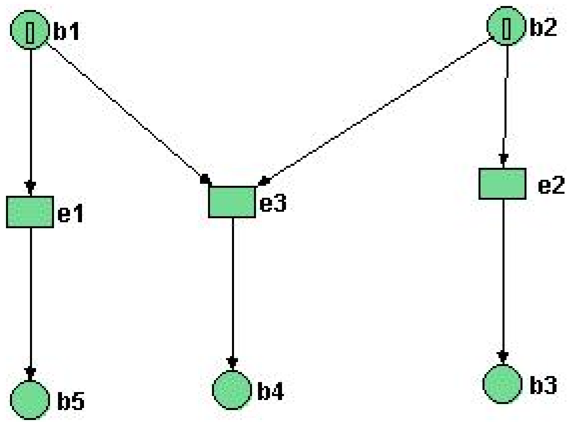

Figure 15. Illustration of confusion.

(C, e_1, e_2) is a c_d confusion that is symmetric.

C = {b_1,b_2}

(C,e_1,e_2)	C[e_2>{b_1,b_3}=C_2
cfl(e_1,C) = {e_3}	
cfl(e_1,C_2) = ∅	

(C,e_2,e_1)	C[e_1>C_2={b_2,b_5}
cfl(e_2,C) = {e_3}	
cfl(e_2,C_2) = ∅	

4. The confusion(C, e_1, e_2) for system is a symmetric confusion that is neither a c_i confusion nor a c_d confusion.

Remark:

C_d Confusions are always symmetric.