Zadanie 1.

a) stałą C obliczamy z warunku:

$$\int_{-\infty}^{+\infty} f(x)dx = 1$$

obliczamy jako sumę całek w poszczególnych przedziałach:

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{-1} 0 + \int_{-1}^{0} 3x^2 dx + \int_{0}^{1} Cx^7\, dx + \int_{1}^{+\infty} 0 = 0 + \left[x^3\right]_{-1}^{0} + C\left[\frac{x^8}{8}\right]_0^1 = 1 + C\frac{1}{8}$$

A zatem powinien być spełniony warunek:

$$1 + \frac{C}{8} = 1$$

$$C = 0$$

dystrybuante ze wzoru: $F(x) = \int_{-\infty}^{x} f(t)dt$

dla $x < -1$ mamy: $F(x) = \int_{-\infty}^{x} f(t)dt = \int_{-\infty}^{x} 0dt = 0$

dla $-1 < x \le 0$ mamy: $F(x) = \int_{-\infty}^{x} f(t)dt = \int_{-\infty}^{-1} 0 + \int_{-1}^{x} 3t^2 dt = 0 + \left[t^3\right]_{-1}^{x} = x^3 - (-1)^3 = x^3 + 1$

dla $x \ge 0$ mamy: $F(x) = \int_{-\infty}^{x} f(t)dt = \int_{-\infty}^{-1} 0 + \int_{-1}^{0} 3t^2 dt + \int_{0}^{x} 0 = 0 + 1 + 0 = 1$

b) $EX = \int_{-\infty}^{+\infty} x \cdot f(x)dx = \int_{-\infty}^{-1} x \cdot 0 + \int_{-1}^{0} x \cdot 3x^2 dx + \int_{0}^{+\infty} x \cdot 0 = 3\int_{-1}^{0} x^3 dx = 3\left[\frac{x^4}{4}\right]_{-1}^{0} = -\frac{3}{4}$

$EX^2 = \int_{-\infty}^{+\infty} x^2 \cdot f(x)dx = \int_{-\infty}^{-1} x^2 \cdot 0 + \int_{-1}^{0} x^2 \cdot 3x^2 dx + \int_{0}^{+\infty} x^2 \cdot 0 = 3\int_{-1}^{0} x^4 dx = 3\left[\frac{x^5}{5}\right]_{-1}^{0} = \frac{3}{5}$

$VarX = EX^2 - (EX)^2$

Zadanie 2.

$$\left.VarX = \frac{(4-a)^2}{12} = 0.75\right\} \Rightarrow a = 1$$

$$EX = \frac{1+4}{2} = 2.5$$

$$P(X > EX \mid X < 3) = \frac{P(2.5 < X < 3)}{P(X < 3)} = \frac{F(3) - F(2.5)}{F(3)} = \frac{\frac{2}{3} - \frac{1}{2}}{\frac{2}{3}} = \frac{1}{4}$$

Zadanie 3.

Korzystamy z twierdzenia: jeśli $X \sim N(\mu, \sigma), Y = aX + b$,to $Y \sim N(a\mu + b, \sqrt{a^2\sigma^2})$

$$EX = \mu = 22,\ VarX = \sigma^2 = 16 \Rightarrow X \sim N(22,4)$$

$$\begin{matrix} EY = a\mu + b = -16 \\ VarY = a^2\sigma^2 = 144 \end{matrix} \quad => \quad Y \sim N(-16,12) \quad => \quad Z = \frac{Y+16}{12} \sim N(0,1)$$

$$P(-20 < Y < 5) = P\left(\frac{-20+16}{12} < Z < \frac{5+16}{12}\right) = \Phi\left(\frac{7}{4}\right) - \Phi\left(-\frac{1}{3}\right) = \Phi(1.75) - 1 + \Phi(0.33)$$