Zadanie 1.

a) stałą C obliczamy z warunku:

$$\int_{-\infty}^{+\infty} f(x)dx = 1$$

obliczamy jako sumę całek w poszczególnych przedziałach:

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{-1} 0 + \int_{-1}^{0} (x^4 + x^2)dx + \int_{0}^{\pi/2} C\sin xdx + \int_{1}^{+\infty} 0 = \frac{1}{5} + \frac{1}{3} + C$$

A zatem powinien być spełniony warunek:

$$\frac{1}{5} + \frac{1}{3} + C = 1$$

$$C = \frac{7}{15}$$

dystrybuante ze wzoru: $F(x) = \int_{-\infty}^{x} f(t)dt$

dla $x < -1$ mamy: $F(x) = \int_{-\infty}^{x} f(t)dt = \int_{-\infty}^{x} 0dt = 0$

dla $-1 < x \le 0$ mamy:

$$F(x) = \int_{-\infty}^{x} f(t)dt = \int_{-\infty}^{-1} 0 + \int_{-1}^{x} (t^4 + t^2)dt = 0 + \left[\frac{1}{5}t^5 + \frac{1}{3}t^3\right]_{-1}^{x} = \frac{1}{5}x^5 + \frac{1}{3}x^3 + \frac{8}{15}$$

dla $0 < x \le \frac{\pi}{2}$ mamy:

$$F(x) = \int_{-\infty}^{x} f(t)dt = \int_{-\infty}^{-1} 0 + \int_{-1}^{0} (t^4 + t^2)dt + \int_{0}^{x} (\frac{7}{15}\sin t)dt = \frac{8}{15} + \frac{7}{15}(1 - \cos x) = 1 - \frac{7}{15}\cos x$$

dla $x > \frac{\pi}{2}$ mamy: $F(x) = 1$

b) $EX = \int_{-\infty}^{+\infty} x \cdot f(x)dx = \int_{-\infty}^{-1} x \cdot 0 + \int_{-1}^{0} x(x^4 + x^2)dx + \int_{0}^{\pi/2} \frac{7}{15} x\sin xdx + \int_{1}^{+\infty} x \cdot 0 = -\frac{5}{12} + \frac{7}{15}$

Zadanie 2.

$$EX = \frac{2}{5} \Rightarrow \lambda = \frac{5}{2}$$

$$VarX = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$EX^2 = VarX + (EX)^2 = \frac{8}{25}$$

$$P(X < EX^2 \mid X > VarX) = \frac{P(4/25 < X < 8/25)}{P(X > 4/25)} = \frac{F(8/25) - F(4/25)}{1 - F(4/25)} = \frac{e^{-2/5} - e^{-4/5}}{e^{-2/5}}$$

Zadanie 3.

$$X \sim N(6,2) \quad => \quad Z = \frac{X-6}{2} \sim N(0,1)$$

$$P(X > a) = P\left(Z > \frac{a-6}{2}\right) = 1 - \Phi\left(\frac{a-6}{2}\right) = 0.4$$

$\Phi\left(\frac{a-6}{2}\right) = 0.6$ Z tablic mamy $\Phi(0.255) = 0.6$

$$\frac{a-6}{2} = 0,255 \quad \Rightarrow \quad a = ...$$