

## Three properties of Petri nets: Reversibility, Boundedness, and Liveness

### 1. REVERSIBILITY of Petri net with initial marking $(N, m_0)$ :

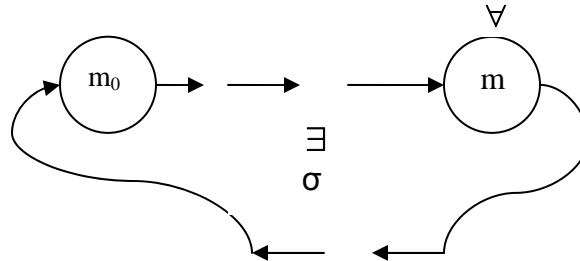


Figure 1. Illustration of the definition of reversibility of Petri net.

Petri net  $N$  with initial marking  $(N, m_0)$  is reversible  $\Leftrightarrow$

$$\forall m \in \text{RG}(m_0) \quad \exists \sigma \quad \text{s.t.} \quad m \xrightarrow{\sigma} m_0$$

**Fact:** From every reachable marking  $m \in \text{RG}(N, m_0)$  there is a sequence of actions that reverses the sequence that lead us to  $m$ .

### 2. BOUNDEDNESS of Petri net with initial marking $(N, m_0)$ :

$(N, m_0)$  is **bounded** if all the places are bounded.

Place  $P$  is **bounded** in  $(N, m_0)$  if and only if  $m(p) < \infty$

### 3. LIVENESS of Petri net with initial marking $(N, m_0)$ :

Net  $(N, m_0)$  is **live** if all transitions are live. Transition  $t$  is **live** in  $(N, m_0)$  if and only if

$$\forall m \in \text{RG}(m_0) \quad \exists \sigma \quad \text{s.t.} \quad m \xrightarrow{\sigma t} m'$$

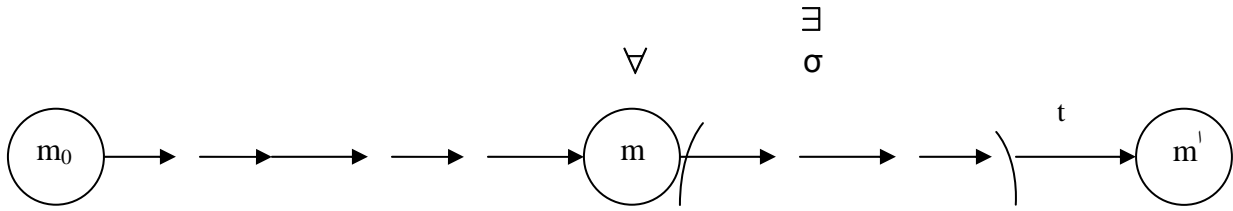


Figure 2. Illustration of the definition of live transition.

#### 4. A HOME STATE:

$m \in \text{RG}(N, m_0)$  is a **home state**  $\Leftrightarrow$

$$\forall m' \in \text{RG}(m_0) \exists \sigma \text{ s.t. } m' \xrightarrow{\sigma} m$$

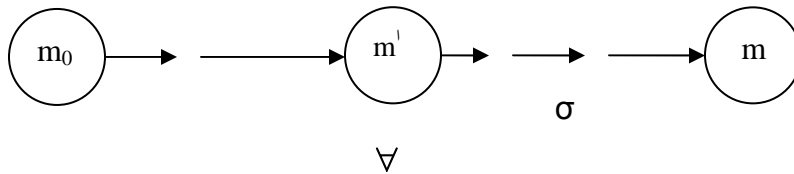


Figure 3. Illustration of the definition of home state.

#### THE LOAN PROTOCOL:

$L = \text{apply. (est1. grant + est2. refuse + est3. (grant + refuse))}$

Abstracting from estimate:

$$\bar{L} = \text{apply. (\tau. grant + \tau. refuse + \tau. (grant + refuse))}$$

Omitting  $\tau$ :

$$\bar{L} = \text{apply (grant + refuse + (grant + refuse))} \equiv$$

apply. (grant + refuse)

different moments of choice

External choice  $a + b$  (influenced from outside organization)

Internal choice  $\tau. a + \tau. b$  (decision taken within organization)

↓

Importance of where the point is taken