CIS 525 Parallel and Distributed Software Development

REDUCTIONS OF OCCURANCE GRAPH FOR COLORED PETRI NETS

Good CP nets simulator $\hat{=}$ good parallel program debugger

Simulation $\hat{=}$ understanding and debugging of a CP net & validation of a large parallel/distributed software system.

What are formal methods in the CP net analysis:

a> Occurrence graphs

b> Place/transition invariants

c> Reduction rules

d> Performance analysis

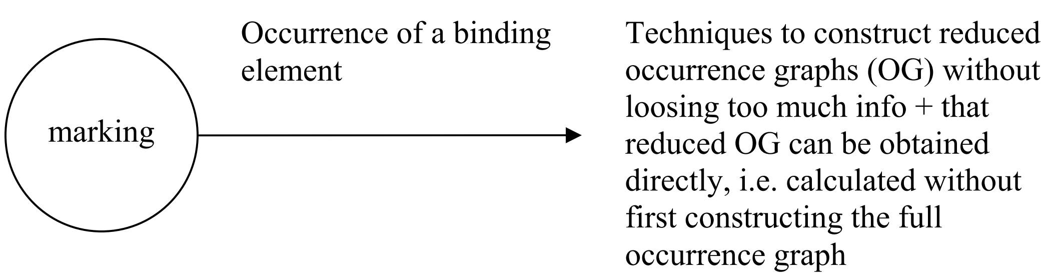


Figure 1. Construction OG through inspection of occurrence set.

REDUCTION 1:

<u>Symmetrical markings</u>: example - database system:

1.

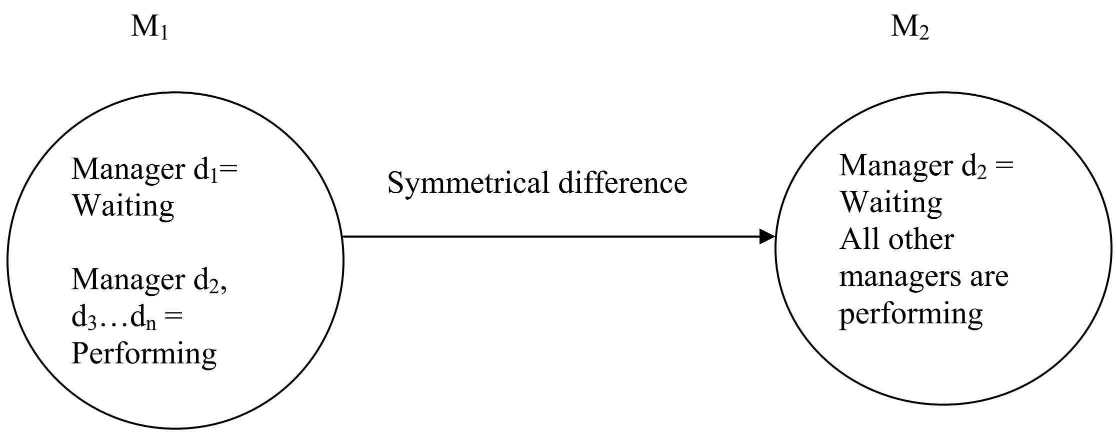


Figure 2. Illustration of markings' symmetry in distributed database.

Symmetry of managers: d_1, d_2, d_3, d_4

2. Colors d_2, d_3, d_4 appear in M_1 in a symmetrical way.

3. Bindings $(SA, <s = d_1, r = d_2 >) = A$

$(SA, <s = d_1, r = d_3 >) = B$

$(SA, <s = d_1, r = d_4 >) = C$

are all enabled in M_1

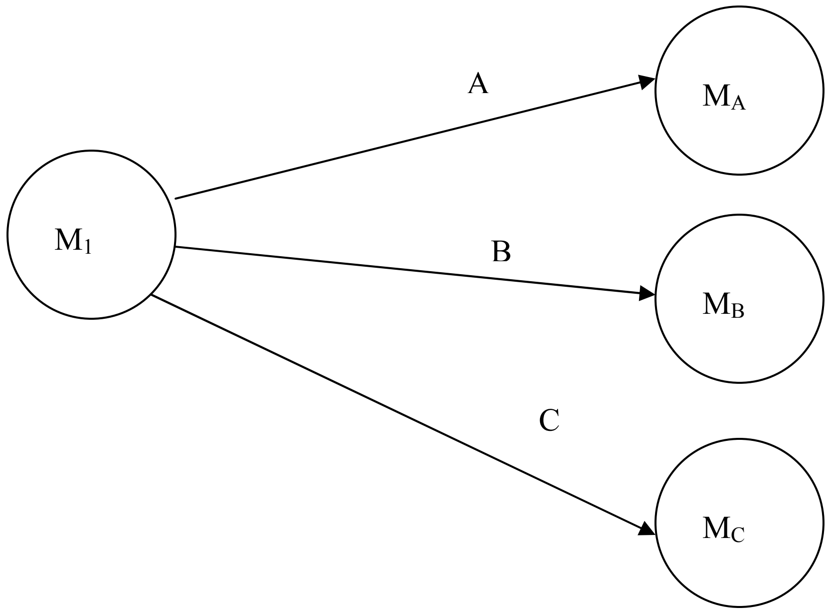


Figure 3. Illustration of OG and enabling of transitions.

(if M_1 is symmetrical) AND (A, B, C are symmetrical) THEN M_A, M_B, M_C are also symmetrical

Theoretical foundations of reduced Occurrence Graphs (OG):

1. Algebraic groups
2. Equivalence classes

Reduced graph $\hat{=}$ folded version of OG

Database example:

	Reduced	Full
Nodes	1+ n*(n+1)/2	$1 + n*3^{n-1}$
Arcs	2*(n-1)*n	$2n + 2*(n-1)\ n*3^{n-2}$

REDUCTION 2:

Stubborn sets:

Observation: A CP net often has a number of occurrence sequences where the steps are identical; except for the order in which they occur.

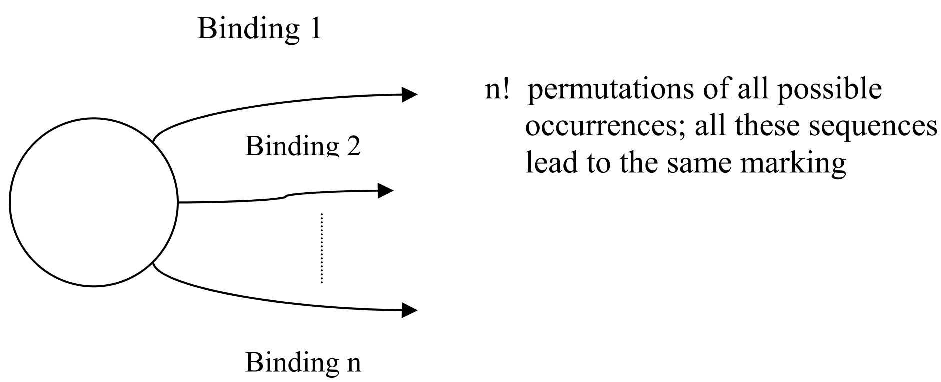


Figure 4. Exponential growth of OG without considering equivalence of some sequences

Facts:

1. For each reachable set one builds a stubborn set.
2. Use of stubborn sets gives a significant reduction, especially for situations when modeled system is composed of many relating independent processes.
3. Stubborn set method requires sometimes construction of several different occurrence graphs.

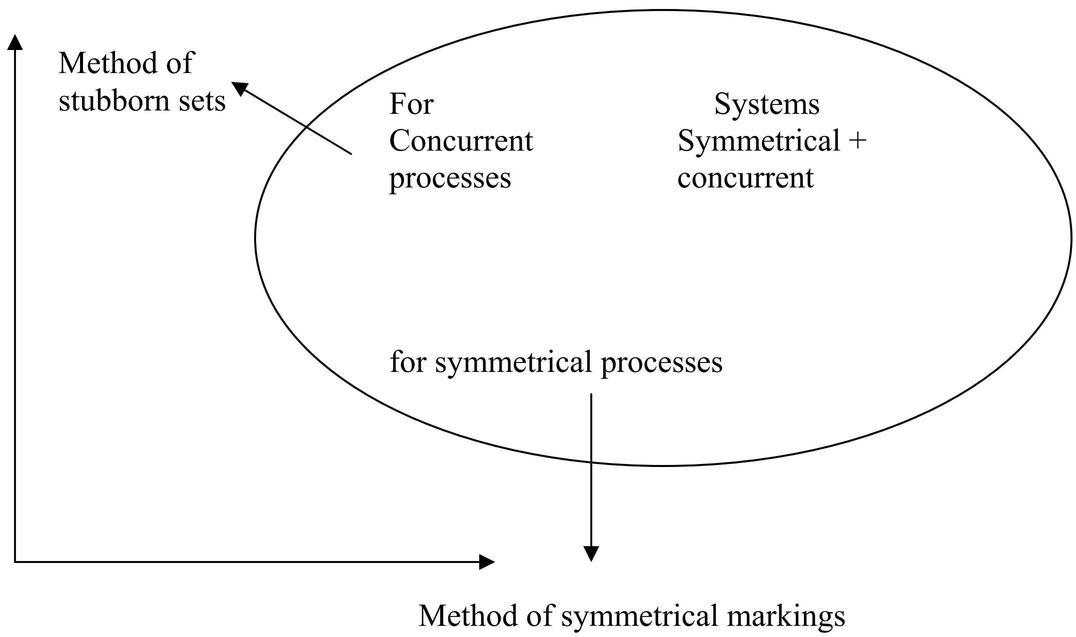
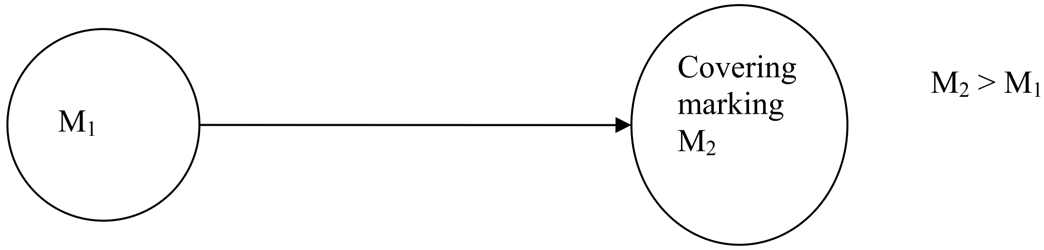


Figure 5. Representation of methods of stubborn sets and symmetrical markings.

REDUCTION 3:



- for CP-nets with finite color sets it can be proven that reduction by means of covering markings always yields a finite occurrence graph.
- this method gives only reduction for unbounded systems (the most practical are bounded systems).
- so much information is lost that liveness and reachability are no longer fully decidable.

Which properties can be answered about bounded systems based on OG:

1. Deadlocks
2. Reachability
3. Marking bounds

And by constructing the strongly connected components of OG one can decide:

1. Liveness
2. Home markings.

REDUCTION RULES

1. Soundness of reduction rules:
   The rules never change the set of properties which we are investigating.
2. Powerful set of reduction rules, i.e. it must yield a significant reduction.
3. Many reduction methods are non-constructive, i.e. the absence of a property in the reduced net does not tell much about why the property is absent in the original net.

PLACE AND TRANSITION INVARIANTS

Idea:

1. Invariants are equations that are satisfied for all reachable markings.
2. Transition invariants are the duals of place invariants, i.e. we attach a weight to each transition. Intuitively, transition invariants characterize a set of occurrence sequences that have no total effect, i.e. have the same start and end marking.

What are gains of place and transition invariants?

1. It is possible to obtain an invariant for a hierarchical CP-net by composing invariants of the individual pages (i.e. it is much easier to use invariants for large systems without encountering the same kind of complexity problems as we have for OG).

2. One can find invariants without fixing the system parameters and hence we can obtain general properties which are independent of system parameters.

3. One can construct the invariants during the design of a system which can lead to improved deign.