AC Linear Circuit Analysis

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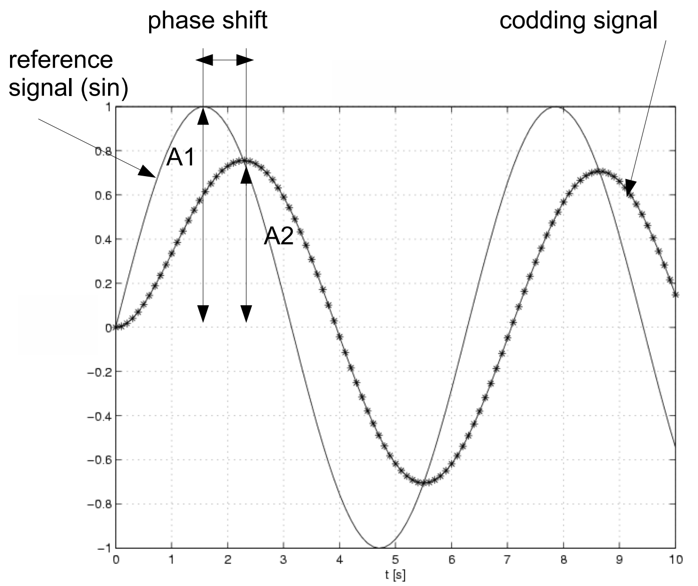
Linearity

If cause and effect are linearly related, the total effect due to several causes acting simultaneously is equal to the sum of the individual effects due to each of the causes acting one at a time.

In the case of AC analysis: the total effect of different frequencies causes acting simultaneously is equal to the sum of the individual effects of frequency causes acting one.

AC signal codding for single frequency f



All sinusoidal signals can be codded with complex numbers as

$$|A|e^{j\cdot\phi} \quad (1)$$

where $|A|$ is an signal amplitude and ϕ phase shift (related to the one, arbitrary chosen signal).

Kirchhoff's Current Law in the Complex Domain

$$\sum_k I_k = 0 \tag{2}$$

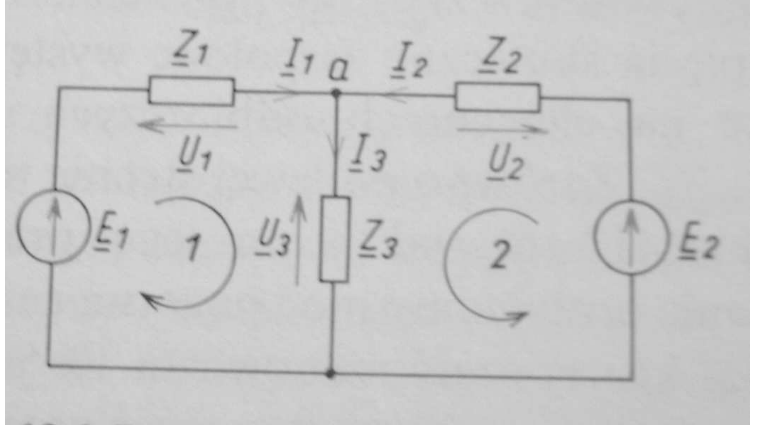
where I_k is an complex number and is described by current amplitude A_k and phase ϕ_k: $I_k = |A_k|e^{j\cdot\phi_k}$.

Kirchhoff's Voltage Law in complex domain

$$\sum_k V_k = 0 \tag{3}$$

where V_k is an complex number and is described by voltage amplitude A_k and phase ϕ_k: $V_k = |A_k|e^{j\cdot\phi_k}$.

AC circuit to analyze



- E_1 is an AC voltage source:
  (sine wave with amplitude $1V$ and pulsation $\omega = 1\frac{rad}{s}$)
- E_2 is an AC voltage source:
  (cosine wave with amplitude $2V$ and pulsation $\omega = 1\frac{rad}{s}$)
- Z_1 in an capacitor with capacity $1F$
- Z_2 is an inductor with conductivity $2H$
- Z_3 is an resistor with resistance 1Ω

Codding AC circuit

- $E_1 = |A|e^{j\cdot 0^\circ} = 1e^{j0^\circ} = 1$ - we have taken E_1 as reference.
- $E_2 = |A|e^{j\cdot 90^\circ} = 2e^{j90^\circ} = j2$
- $Z_1 = \frac{1}{j\omega \cdot C} = \frac{1}{j\cdot 1\cdot 1} = -j$
- $Z_2 = j \cdot \omega L = j \cdot 1 \cdot 2 = j2$
- $Z_3 = 1$

Mesh Analysis

For 2 nodes we write 2 - 1 equations. For node ”a“:

$$I_3 = I_1 + I_2 \tag{4}$$

For each mesh (2 meshes) we write equation:

$$\begin{cases} E_1 = Z_1 I_1 + Z_3 I_3 \\ E_2 = Z_2 I_2 + Z_3 I_3 \end{cases} \tag{5}$$

and solve.

We can write equations (4) and (5) as follow:

$$\begin{cases} I_3 = I_1 + I_2 \\ 1 = -j \cdot I_1 + 1 \cdot I_3 \\ j2 = j2 \cdot I_2 + 1 \cdot I_3 \end{cases} \tag{6}$$

with solution:

$$\begin{cases} I_1 = \frac{2}{5} - j\frac{1}{5} \\ I_2 = \frac{4}{5} + j\frac{3}{5} \\ I_3 = \frac{6}{5} + j\frac{2}{5} \end{cases} \tag{7}$$

Node analysis

Let denote potential of node ”a“ with V_a. Currents I_3, I_1 and I_2 in formula (4) can be expressed as:

$$\begin{cases} I_1 = \frac{E_1 - V_a}{Z_1} \\ I_2 = \frac{E_2 - V_a}{Z_2} \\ I_3 = \frac{V_a}{Z_3} \end{cases} \tag{8}$$

and for node ”a“ current balance ($I_3 = I_1 + I_2$) is equal

$$\frac{V_a}{Z_3} = \frac{E_1 - V_a}{Z_1} + \frac{E_2 - V_a}{Z_2} \tag{9}$$

For data

$$\frac{V_a}{1} = \frac{1 - V_a}{-j} + \frac{j2 - V_a}{j2} \tag{10}$$

potential is equal

$$V_a = \frac{6}{5} + j\frac{2}{5} \tag{11}$$

and currents

$$\begin{cases} I_1 = \frac{E_1 - V_a}{Z_1} = \frac{2}{5} - j\frac{1}{5} \\ I_2 = \frac{E_2 - V_a}{Z_2} = \frac{4}{5} + j\frac{3}{5} \\ I_3 = \frac{V_a}{Z_3} = \frac{6}{5} + j\frac{2}{5} \end{cases} \tag{12}$$