Passive Signal Processing

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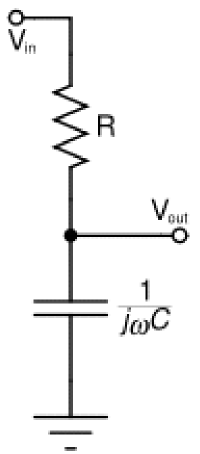
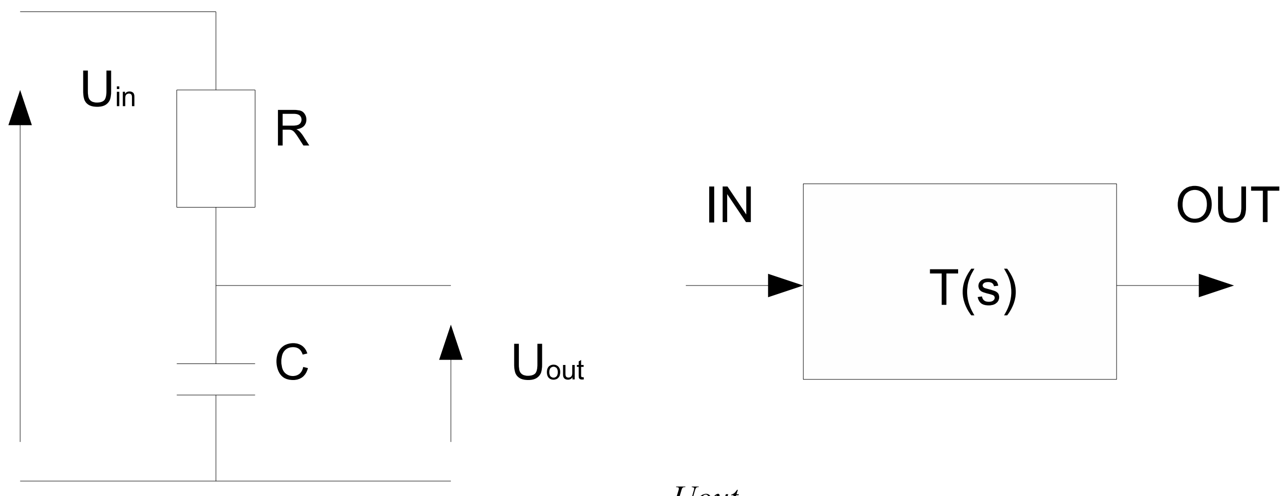
Impedance divider - an example

$$V_{\text{out}} = \frac{Z_2}{Z_1 + Z_2} \cdot V_{\text{in}}$$



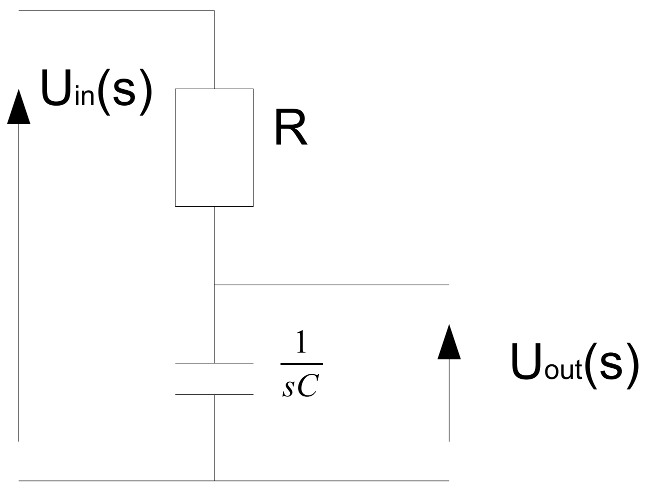
A voltage divider is usually thought of as two resistors, but for electronics signals at a given frequency capacitors, inductors, or any combined impedance can be used.
The ratio contains an imaginary number, and actually contains both the amplitude and phase shift information of the filter. To extract just the amplitude ratio, calculate the magnitude of the ratio, or just use the reactance of the capacitor instead of the impedance.

Impedance divider as a black box



$$T(s)=\frac{Uout}{Uin}$$

Transmittance of RC divider



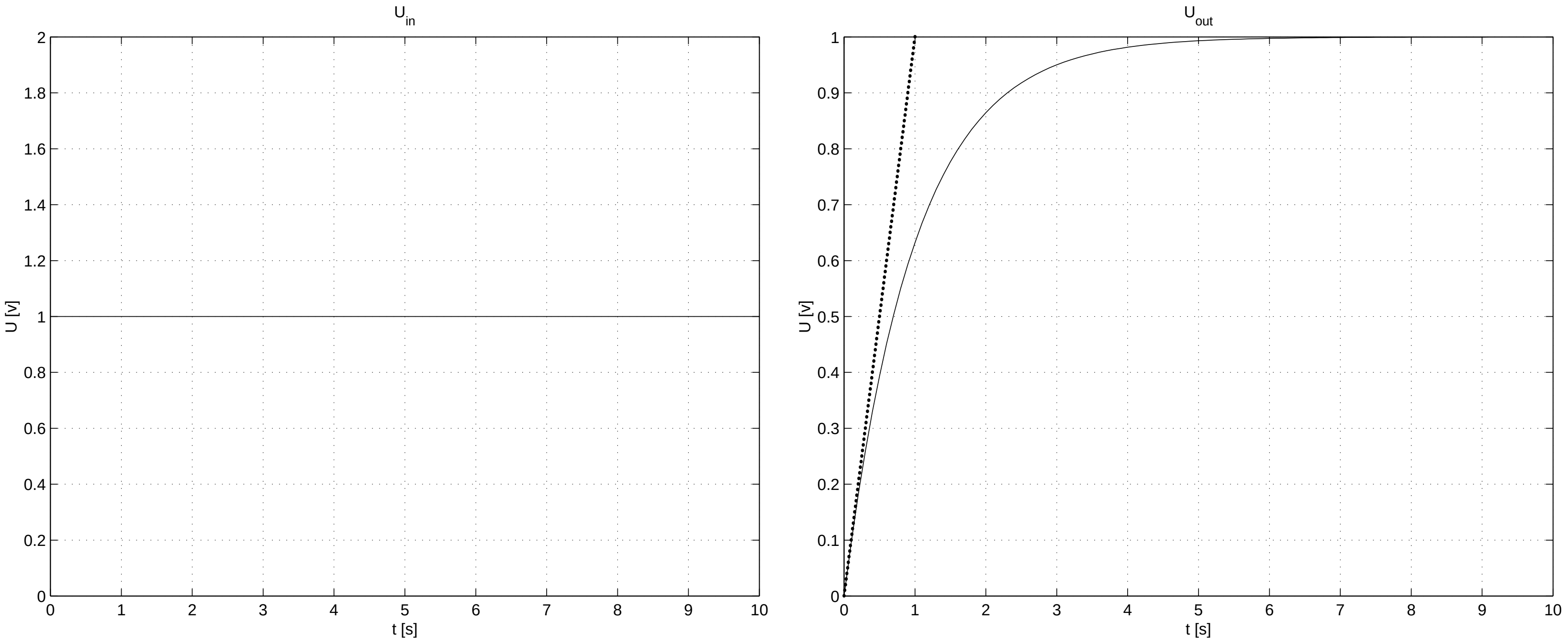
For transfer function:

$$T(s) = \frac{U_{out}(s)}{U_{in}(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + s \cdot RC} = \frac{1}{1 + s \cdot T} \qquad (1)$$

where $T = R \cdot C$ is called *time constant*

Step response of RC divider

Time constant is equal: ($T = RC = 1$)



Time constant

In a capacitor-resistor circuit, the number of seconds required for the capacitor to reach 63.2% of its full charge after a voltage is applied. The time constant of a capacitor with a capacitance (C) in farads in series with a resistance (R) in ohms is equal to $R \cdot C$ seconds.

How *transfer function* $T(j\omega)$ **depends on frequency** f **?**

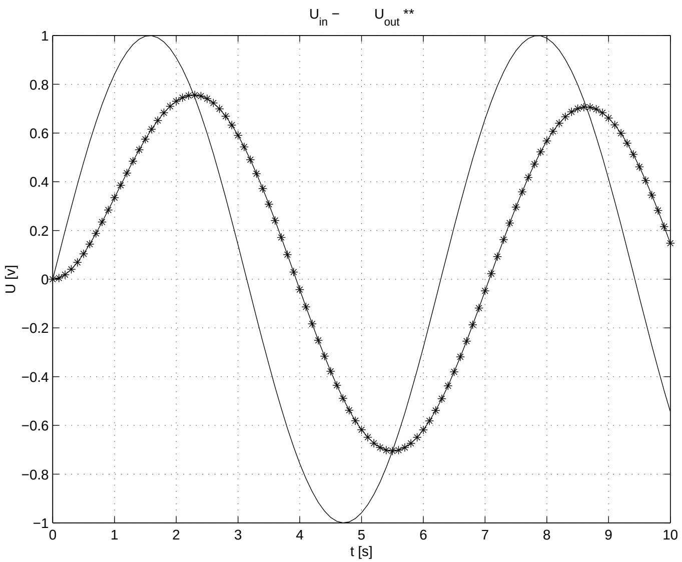
For any frequency *transfer function* is an complex number, and can be describe as

$$T(j\omega) = |Z|e^{j \cdot \varphi} \quad (2)$$

where $|Z|$ is an *magnitude* and φ is and angle (*phase* - in electronic terminology)

Time domain interpretation of *magnitude* and *phase* of transmittance $T(j\omega)$

Time constant $T = RC = 1$, frequency $f = 1\frac{rad}{s}$



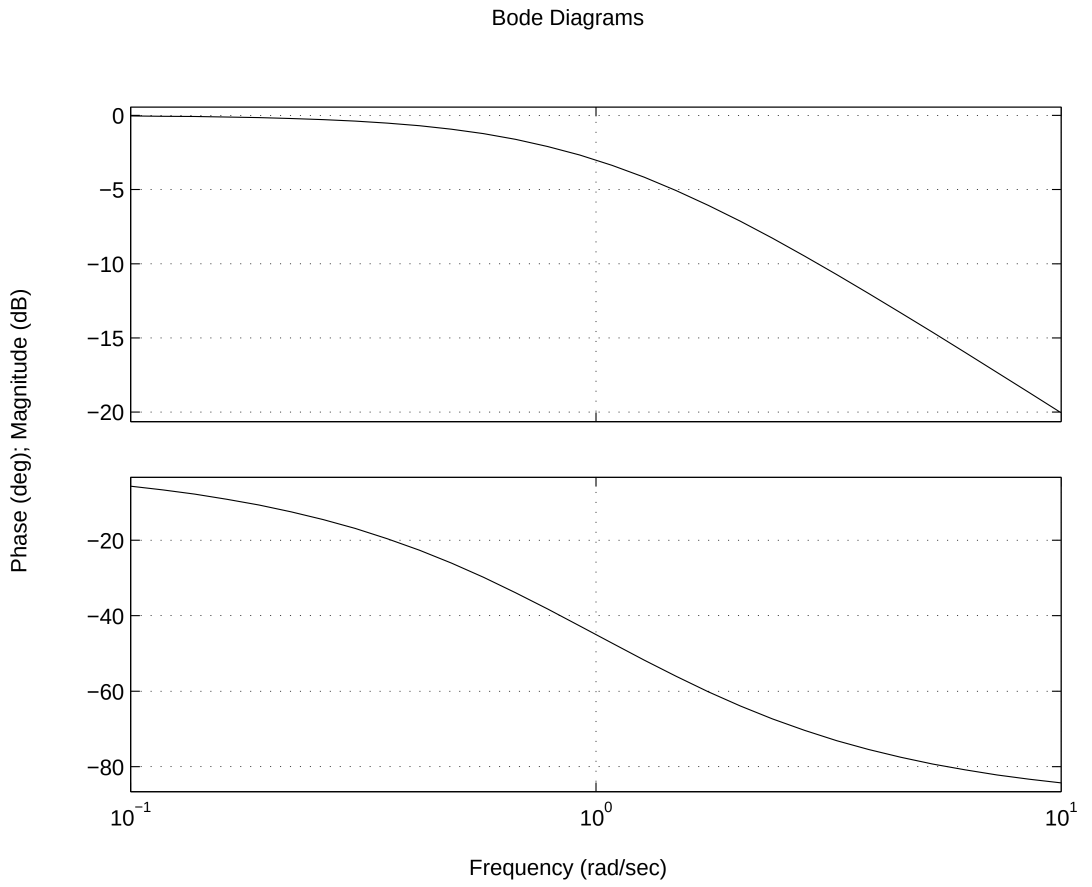
- transmittance amplitude: $|Z| = \frac{AMPLITUDE_{OUT}}{AMPPLITUDE_{IN}}$
- transmittance phase: time shift of output signal expressed in radians (or degree)

Important remarks on linear systems

- Transmittance describes linear object (or linear approximation of object)
- INPUT Sinusoidal signal with frequency f ”generates“ OUTPUT sinusoidal signal with the same frequency f.
- Superposition: If input signal is combination of frequency f_1 and f_2 output signal of *linear system* is superposition (sum) of response for sinusoidal f_1 and f_2.

Magnitude and phase diagrams

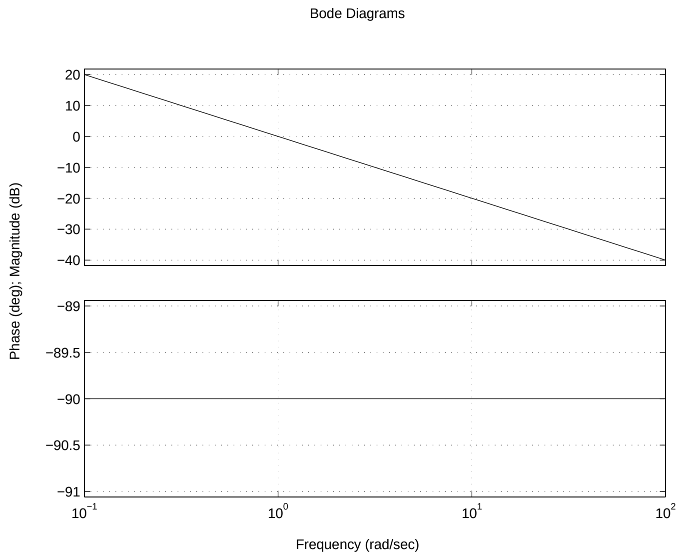
Time constant: $T = RC = 1$



Logarithmic scale of Magnitude

Transmittance of capacitor

$T(S) = \frac{U(S)}{I(s)}$



- Module $|Z|$ can be expressed in logarithmic scale

$$|Z| = 20 \cdot \log_{10}\left(\frac{AMPLITUDE_{OUT}}{AMPPLITUDE_{IN}}\right) \tag{3}$$

unit decybele [dB]

- slope of amplitude curve $-20\frac{dB}{dec}$

Cut-off frequency

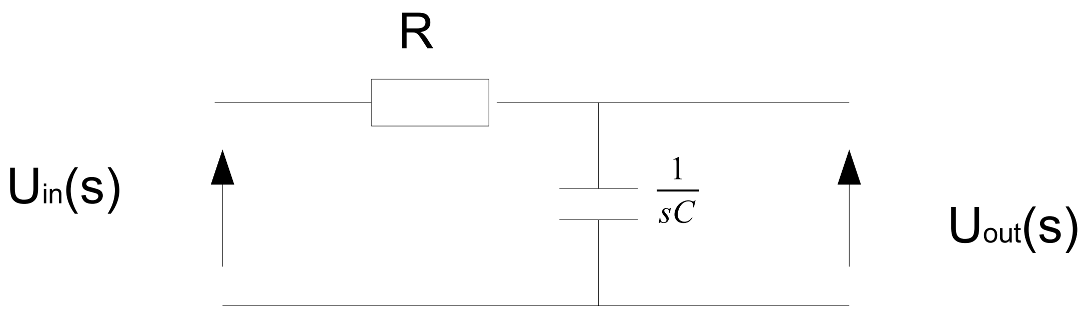
The cut-off frequency is the point at which the magnitude is $\frac{1}{\sqrt{2}} \approx 0.707$. (–3 dB in decybel scale)

$$20 \cdot \log_{10}(\frac{1}{\sqrt{2}}) \approx -3dB \tag{4}$$

when the amplitude decrees $\frac{1}{\sqrt{2}}$ times power decrees $\frac{1}{2}$ times ($P = \frac{U^2}{R}$)

$$10 \cdot \log_{10}(\frac{1}{2}) \approx -3dB \tag{5}$$

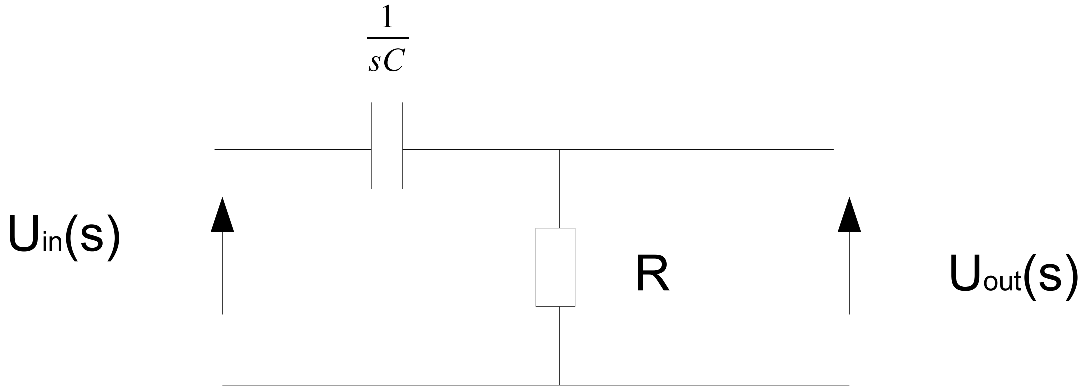
Low-Pass Filters



- *Low-pass filter* is a filter whose passband extends from dc to some finite cut-off frequency.
- RC - divider is low-pass filter
- Cut-off frequency is equal

$$f_{cut} = \frac{1}{2\pi RC} \tag{6}$$

High-Pass Filters

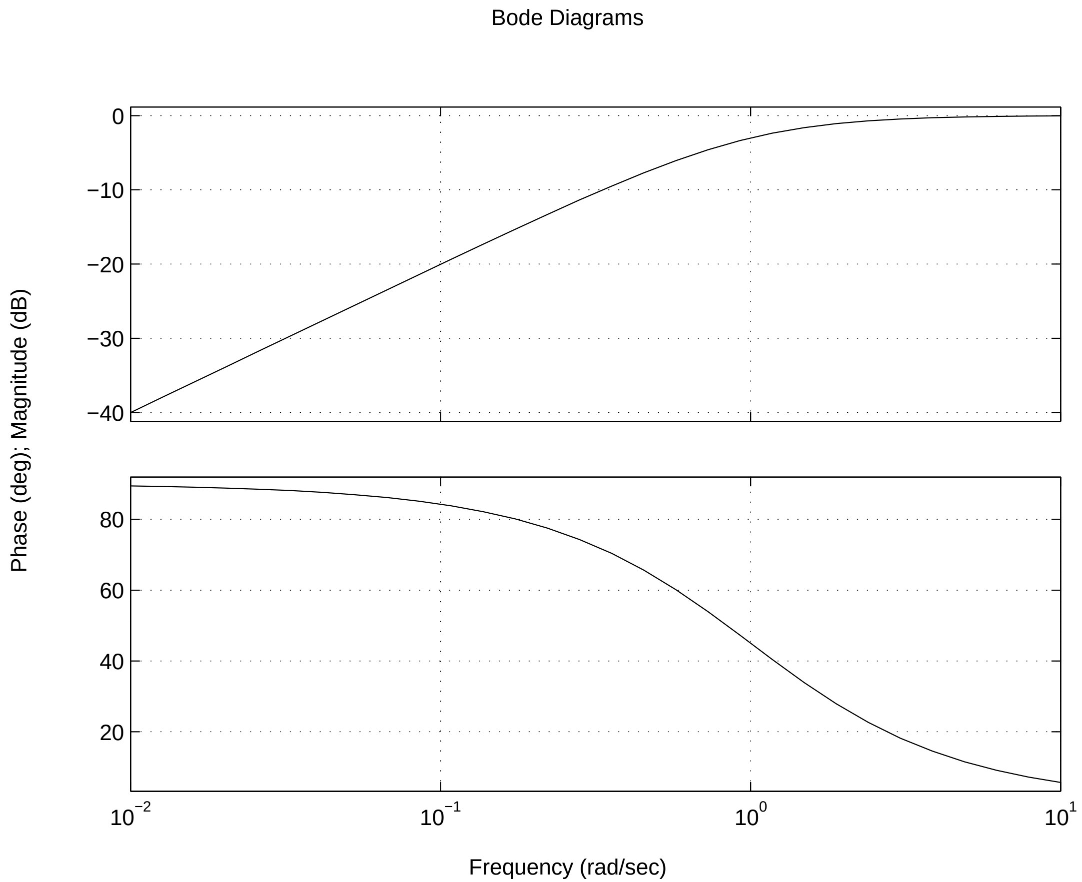


- *High-pass filter* is a filter whose band extends from some finite cut-off frequency to infinity.
- Transfer function of high-pass filter is equal $T(s) = \frac{R}{R+\frac{1}{sC}} = \frac{sRC}{1+sRC}$
- CR - divider is low-pass filter
- Cut-off frequency is equal

$$f_{cut} = \frac{1}{2\pi RC} \tag{7}$$

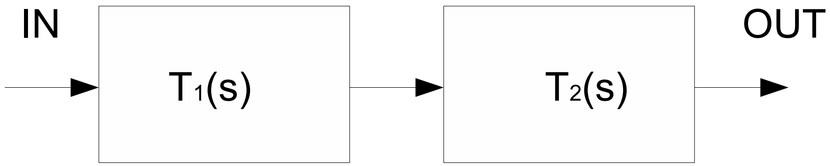
High-pass filter magnitude and phase characteristic

Time constant: $T = RC = 1$



Bandpass filter

- *Bandpass filter* is a filter whose passband extends from a finite lower cut-off frequency to a finite upper cutoff frequency.
- Bandpass filter is an combination of low-pass and high-pass filters

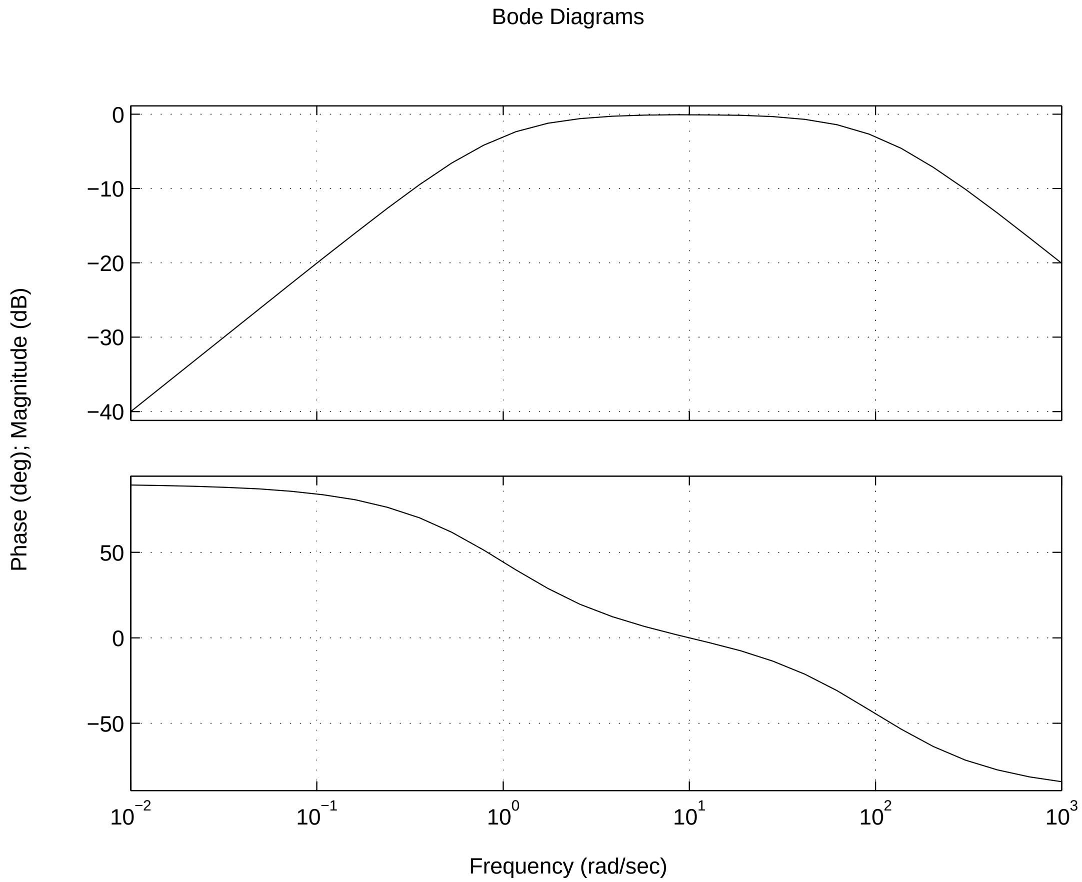


$$T(s)=\frac{Uout}{Uin}=T(s)_1\cdot T(s)_2$$

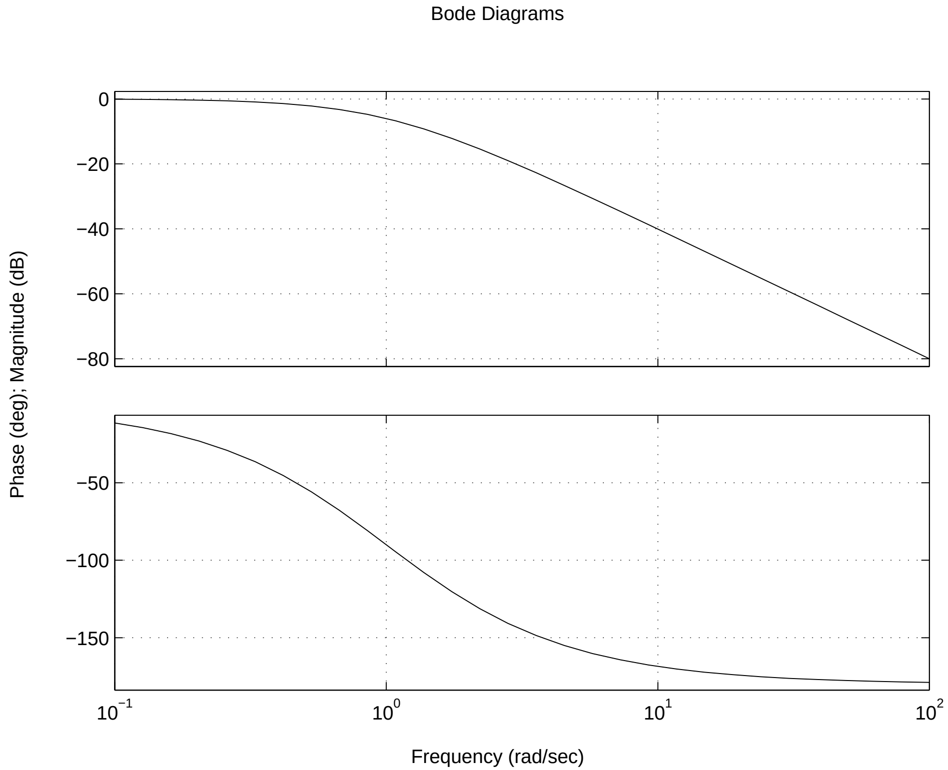
- transfer function $T(s) = \frac{sR_1C_1}{1+R_1C_1} \cdot \frac{1}{1+R_2C_2}$

Bandpass filter magnitude and phase characteristic

Time constants: $T_1 = R_1C_1 = 1, T_2 = R_2C_2 = 0.01$



Higher-order filters



- Slope of amplitude characteristic can be change by connection in serial two or three filters. In that case $-40\frac{dB}{dec}$
- Number of filters is limited to 3 (stability problem - will be discuss later - Lecture 7)