02.03.2011

AUG

1. $\emptyset$ $\{\varepsilon\}$

   0 elementów 1 element

2. $\{\varepsilon, b, bbab\}$
3. $\{abab, abbab, abbbab, babab, babbab\}$
4. $\{ba, baba\}$
5.

Ćw 2. 509 183 249 to prefiksów : sufiksów (w Σ)

51324

519

~~580~~

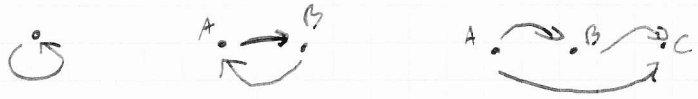
- nie spójnych $\leq 2^9$
- spójnych

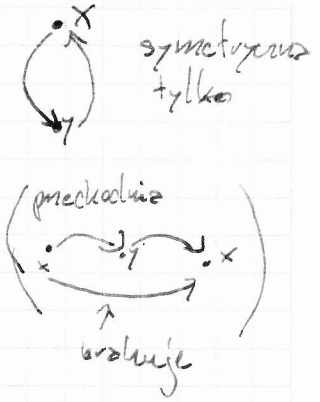
  $\{\varepsilon\}$ $\{5, 0 \ldots\}$

  $1 + 9 + 8+7+\ldots+1 = \frac{9(9+1)}{2}$

- podsłowa które występują ciągu nie więcej niż raz - 9

Ćw 10. zwrotna, symetryczna, przechodnia





- $x \; r \; y$ wtw, gdy $|x| = |y|$

  Z:T S:T P:T równoważności

- $x \; r \; y$ wtw, gdy x jest anagramem y

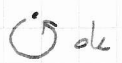
  np. kot, tak, ktoś

  Z:T S:T P:T równoważności

- $A \; r \; B$ wtw, gdy $A^* = B^*$

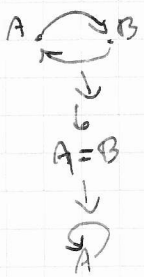
  ↑ z równości wynika że to równoważność (antysymetryczny)

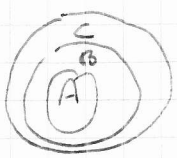
Ćw 11. zwrotna, przechodnia, antysymetryczna



- $A \; r \; B$ wtw, gdy $A \subseteq B$

  Z: T AS: T P: T (porządek częściowy)





- porządek ~~leksykograficzny~~ leksykograficznie

  $\Sigma : T \quad AS : T \quad P : T$

- relację bycia prefiksem

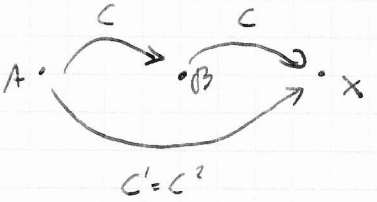
  $\Sigma : T \quad AS : T \quad P : T$

- $A \prec B$ wtw., gdy istnieje takie γ, że $A\gamma = B$

  np. $\gamma = \{\varepsilon\}$

  $\Sigma : T \quad AS : T \quad P : T$

  trzeba by odjąć, a dla $\gamma - \{\varepsilon\}$



~~23.~~ 23.III

9.03.2011

1) $(a(b?)a)?(a[a-c]a)^+ = ((a(b|\varepsilon)a)|\varepsilon)(a(a|b|c)a)(a(a|b|c)a)^*$

2) komórki $[5-8]\,[0-9]$..

stacjonarne $([2-6]\,[0-9] \mid [7-8]\,[1-9])\,\underbrace{[0-9]}_{\times 7}$

3) $(a^*(aba|abb)|b^*(bab|baa))^*(a(a^*ba|a^*bb)|b(b^*ab|b^*aa)) =$

$((a^+ba|a^+bb)|(b^+ab|b^+aa))^*((a^+ba|a^+bb)||(b^+ab|b^+aa)) =$

$((a^+ba|a^+bb)|(b^+ab|b^+aa))^+ = (a^+b\,(a|b))||(b^+a\,(a|b))^+ =$

$=((a^+b|b^+a)(a|b))^+$

Ćw1.4 $b^*(ab^*ab^*)^*$

1.2 np. ABC123456 ↘ $[A-Z]\,[A-Z]\,[A-Z]\,[0-9]\,[0-9]\,..\,[0-9]$

~~1.3~~

1.8 $(b|c|ab|ac)^*(a|\varepsilon)$

1.11 $(a|b)^*|(a|c)^*|(a|c)^*$

1.10 $a^+|b^+|c^+$

1.12 $((a^+b|b^+a)(a|b)^*)\,|\,((b^+c|c^+b)(b|c)^*)\,|\,((a^+\varepsilon|c^+a)(a|c)^*)$

Ćw2.1 $(a?\,b?)^*\;(a|b)^*$

$((a|\varepsilon)(b|\varepsilon))^*$

$(\varepsilon|(a|b)|ab)^*$ — do siebie

dają się wygenerować z $(a|b)^*$

$(a?b?)^* \equiv (a|b)^*$

Ćw. 2.2 $(a|b)^*$ $a^*|b^*$

$L \supset P$ np. ab

Ćw. 2.3 $(a^*b)^*$ $(b^*a)^*$

$L \neq P$

np. b a

2.4 $(a|b)^*$ $(aa|ab|ba|bb)^*$

$L \supset P$ ↓ tylko parzyste wyrazy

2.5 $aa(bbaa|baa)^*$ $(aabb|aab)^*aa$

$L \overset{6m}{\equiv} P$

Ćw 3.1

$((ab^+)?|[x-z])^*$

$((ab(b)^*)?|(x|y|z))^*$

$((ab(b^+)|\varepsilon)|(x|y|z))^*$

3.2 $([a-c]|a(b!))^+$

$((a|b|c)|a(b|\varepsilon))^+$

$((a|b|c)|a(b|\varepsilon))((a|b|c)|a(b|\varepsilon))^*$

$(a|b|c|ab)(a|b|c|ab)^*$

Ćw 4.1 $(a|b)(aa|bb|ba)^*(a|b) =$

$= (a|b)((a|b)(a|b))^*(a|b) =$

$= ((a|b)(a|b))^*((a|b)(a|b)) = ((a|b)(a|b))^+$

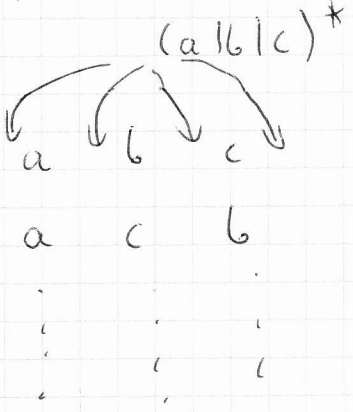
Ex 4.2 $(a(b|\varepsilon)\,|\,ba(b|\varepsilon))^+\,|\varepsilon \equiv$

$(a\,|\,ab\,|\,ba\,|\,bab)^+\,|\varepsilon \equiv$

$((b?)a(b?))^+\,|\varepsilon \equiv$

$((b?)a(b?))^*$

Ex 1.13



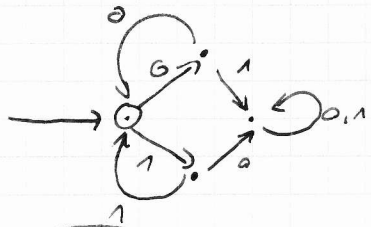
Ex 1.5 $(b|a^+c)^*a^*$

1.5 $a(a|b|c)^*a \;|\; b(\quad)^*b \;|\; c(\quad)^*c \;|\; a \;|\; b \;|\; c$

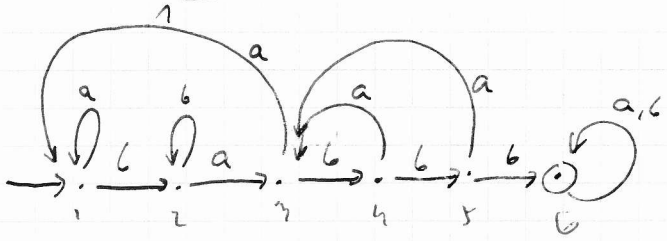
AUG

pondelok

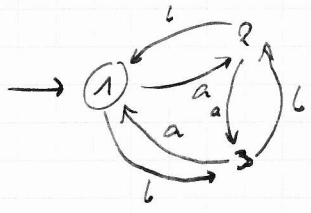
1.



2.

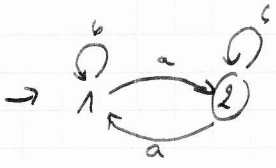


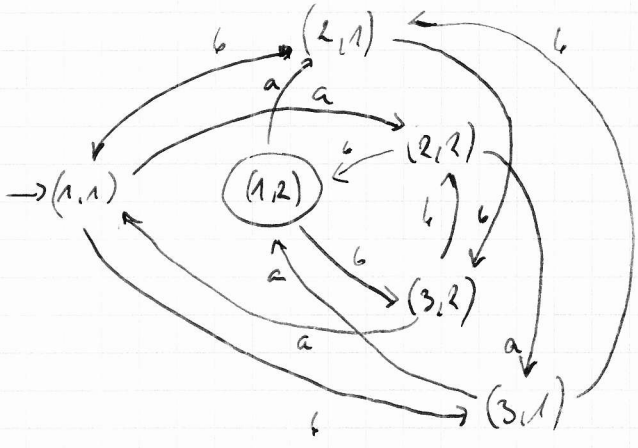
3.



	a	b
→ F 1	2	3
2	3	1
3	1	2

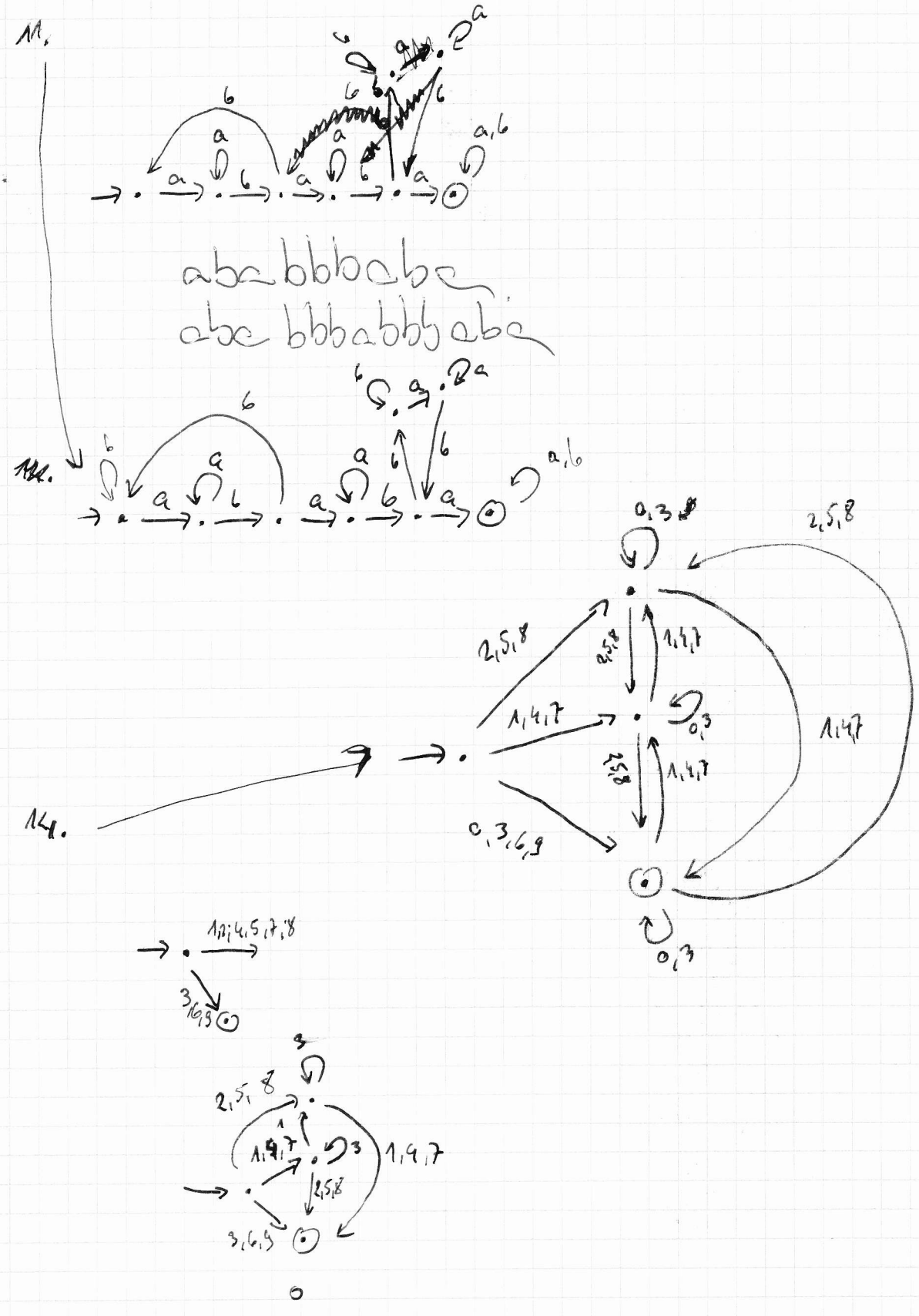
	a	b
→ 1	2	1
F 2	1	2

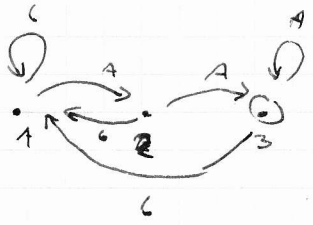




18, 10, 8, 4, 2

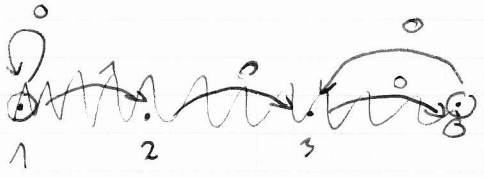
2, 4, 8, 10, 18

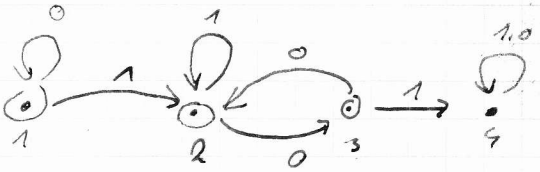

11.
abababbbabc
abcbbbabbbabc
111.
14.
1,4,7
2,5,8
0,3,6,9
0,3
1,4,7
2,5,8
1,4,7
0,3
1,4,7
2,5,8
1,4,7
2,5,8
3,6,9
3
1,4,7
2,5,8
3,6,9



abababcc

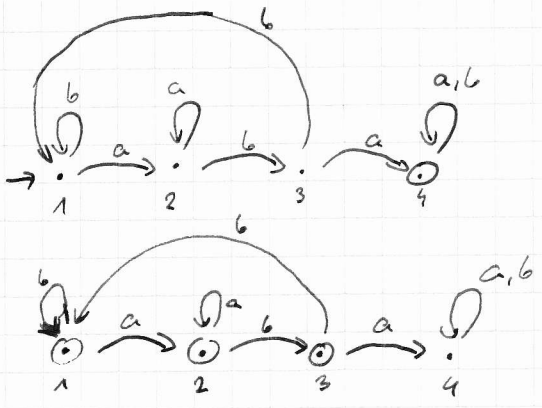
		a	c
→	1	2	1
	2	3	1
	3	3	1





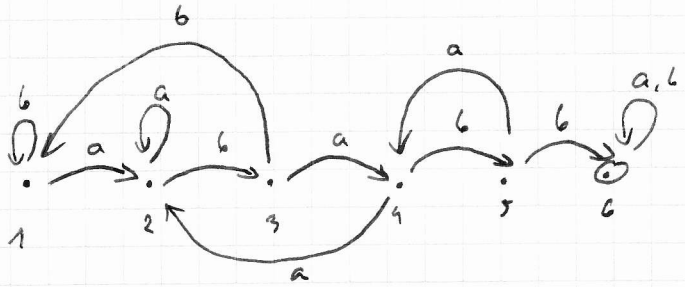
		A	B
→	F 1	2	1
F	2	2	3
F	3	4	2
	4	4	4

8.



		a	b
→	1	2	1
	2	2	3
	3	4	1
	4	4	4

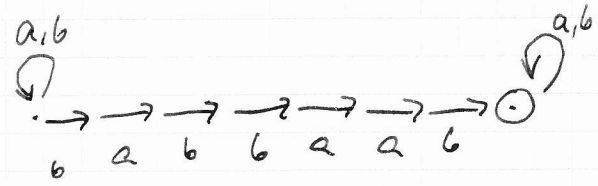
10. ababb



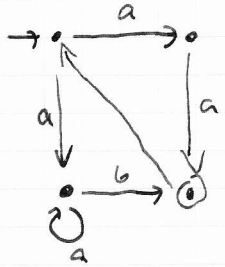
		a	b
→	1	2	1
	2	2	3
	3	4	1
	4	2	5
	5	4	6
	6	6	6

AUC 28.03.2011

praca dom 1. babbaab

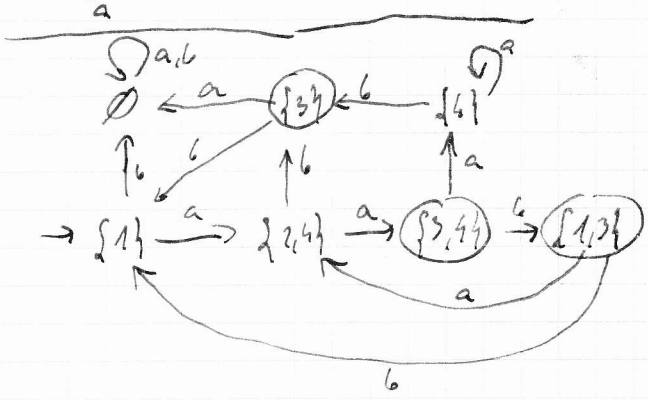


2.



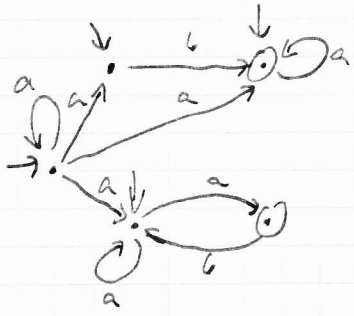
	a	b
→ 1	2,4	–
2	3	–
F 3	–	1
4	4	3

	a	b
→ {1}	{2,4}	∅
{2,4}	{3,4}	{3}
∅	∅	∅
F {3,4}	{4}	{1,3}
F {3}	∅	{1}
{4}	{4}	{3}
F {1,3}	{2,4}	{1}



3.

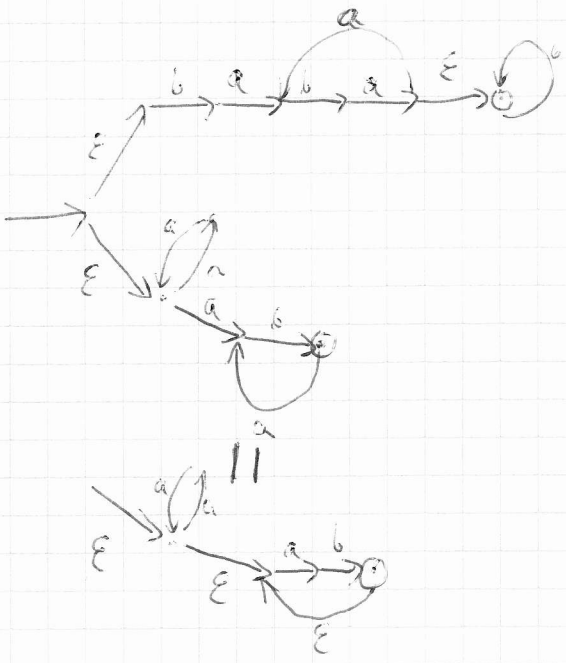
$a^*(b?c^* \mid (ab?)^*a)$



AUG 04.04.2011

grupa
Ștefănuș

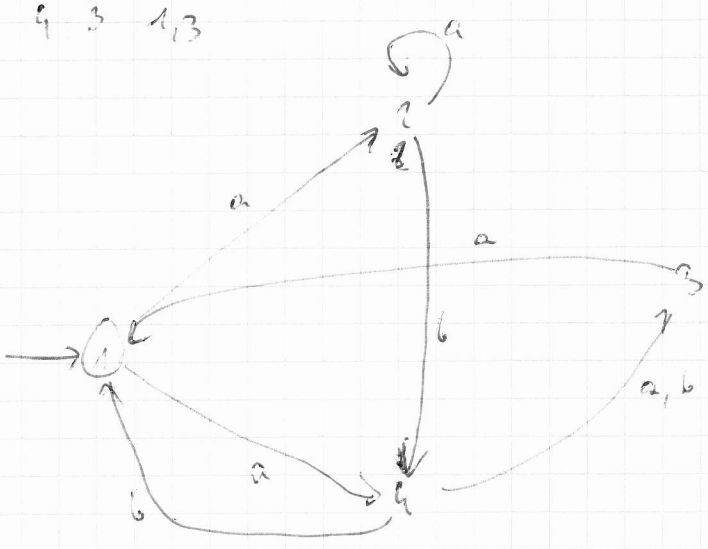
2. $b(aba)^+b^* \mid (aa)^*(ab)^+$



1. a) 1) b
2) d
3) c
4) a

3.

	a	b	
→F 1	2,4		
2	2	4	
3	1		
4	3	1,3	



$$\left(\alpha_{1,1}^{f_{2,3,4}} \right)^*$$

$$\left(\alpha_{1,1}^{g_{2,3,4}} \mid \alpha_{1,1}^{f_{2,3,4}} \left(\alpha_{q,4}^{g_{2,3,4}} \right)^* \alpha_{i,1}^{1_{2,3}} \right)^*$$

$$\left(\varepsilon \mid (a \mid aa^* b)(\varepsilon)^* (b \mid aa \mid b_2) \right)^*$$

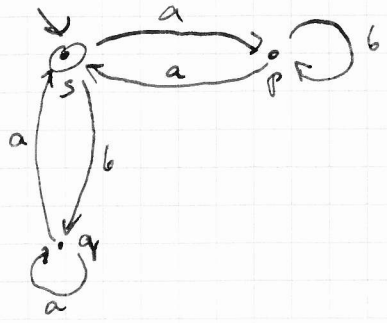
$$((a \mid aa^* b)(b \mid aa \mid b_2)^*$$

AUG śr 06.04.2011 - niedeterministyczne i ε-przejścia

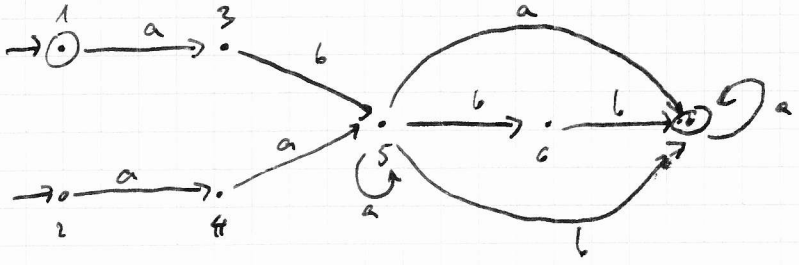
2)

1.

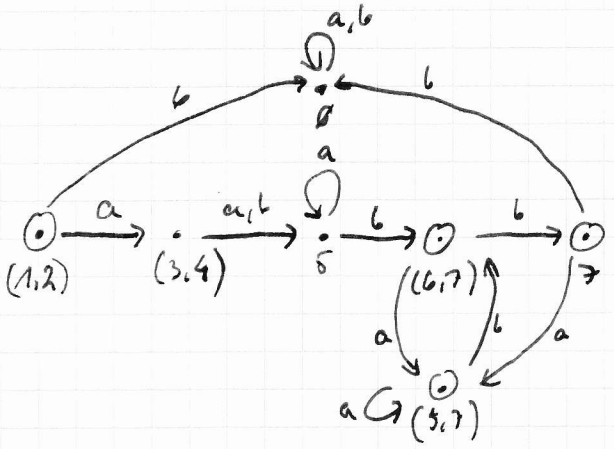
	a	b
→F s	p	q
p	s	p
q	s,q	-



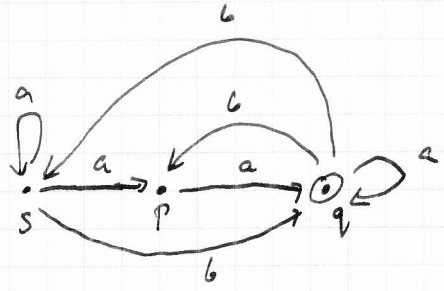
3.



	a	b
→ {1,2}	{3,4}	∅
{3,4}	{5}	{5}
{5}	{5}	{6,7}
{6,7}	{5,7}	{7}
{5,7}	{5,7}	{6,7}
{7}	{5,7}	∅

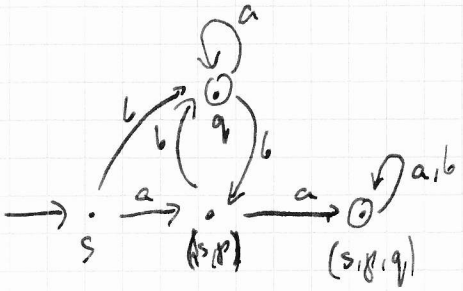


4.

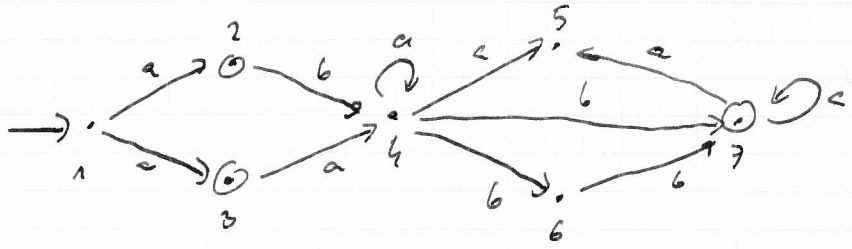


	a	b
→ s	s,p	q
	q	-
F q	q	s,p

	a	b
→ s	s,p	q
F q	q	s,p
s,p	s,p,q	q
F s,p,q	s,p,q	s,p,q

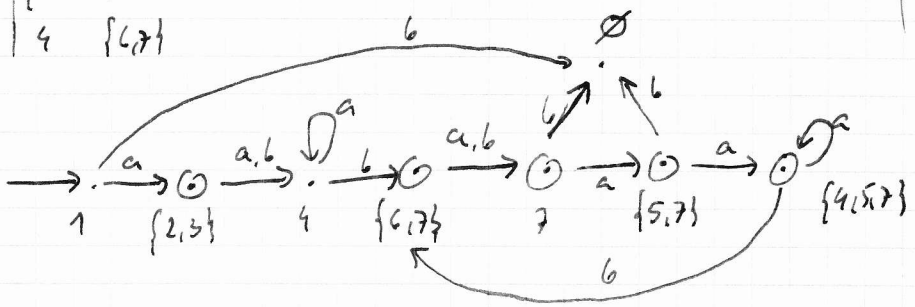


4.

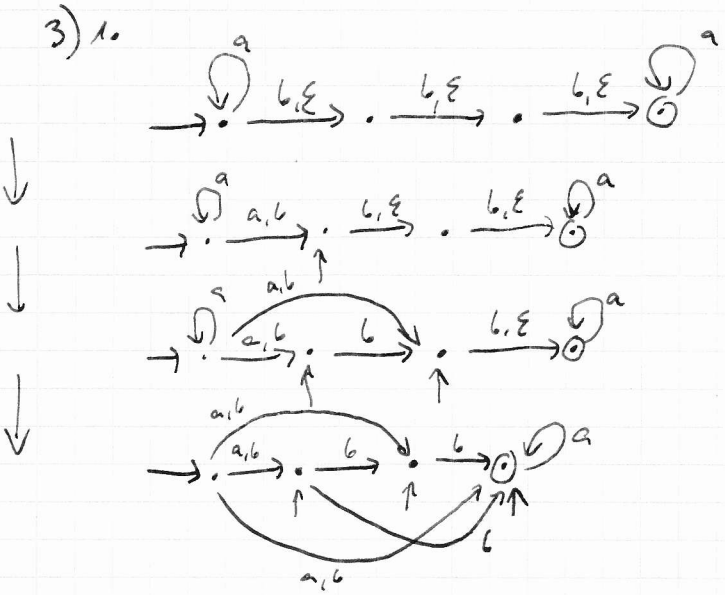


	a	b
→ 1	{2,3}	∅
F {2,3}	{4}	{4}
4	4	{6,7}

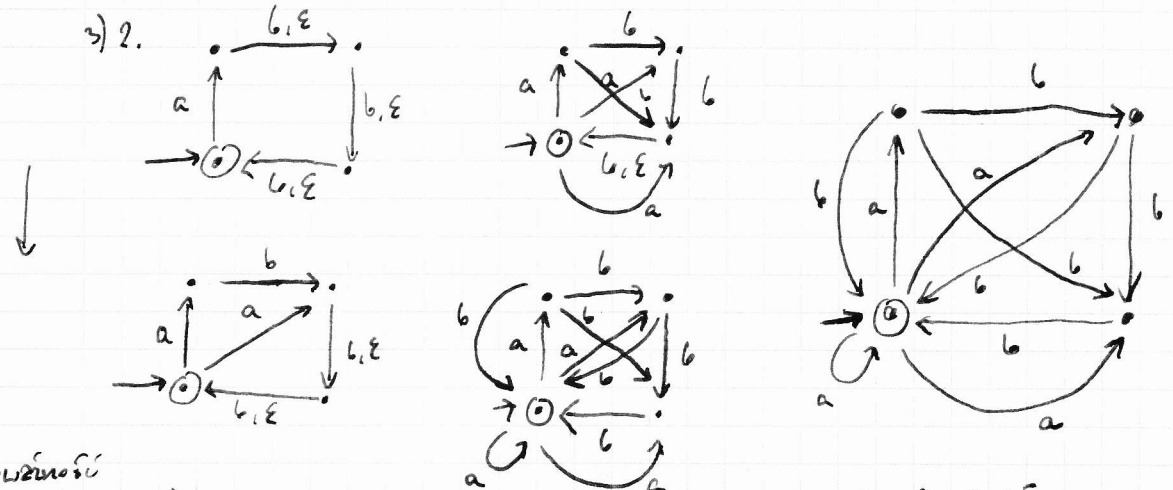
	a	b
F {6,7}	{5,7}	∅
		∅



3) 1.



3) 2.



Řešení:
1) b,d 2) b,d 3) 2,4,6 4) 2,4,6 5) 2,4

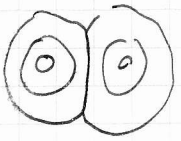
SALA 223 čt 8.45

AlW 4. 11.04.2011

1. $\{a^n b^{n+k} c^k : n, k \geq 1\}$

$a^n b^n b^k c^k$

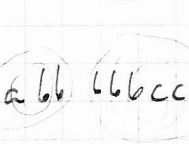
X Y



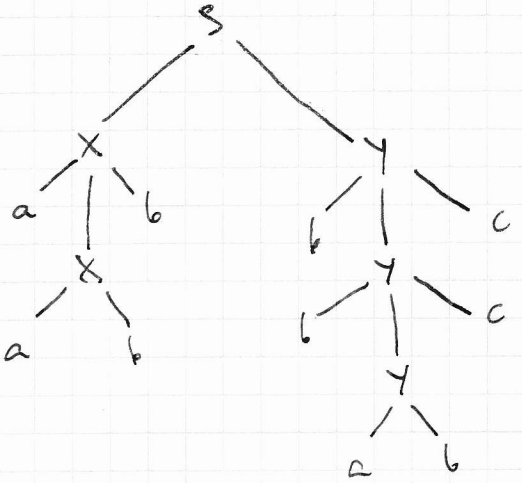
$S \to XY$

$X \to aXb \mid ab$

$Y \to bYc \mid bc$



aabb bbbccc



2.

$ab(b^* \mid a^*)$

liniowa

$S \to abX \mid abY$

$X \to bX \mid \varepsilon$

$Y \to aY \mid \varepsilon$

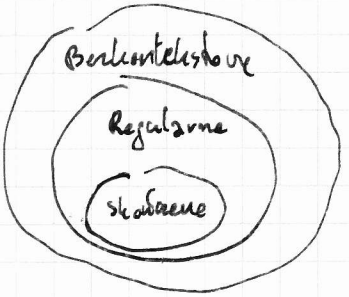
silnie liniowa

$S \to aA$

$A \to bX \mid bY$

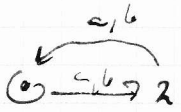
$X \to bX \mid \varepsilon$

$Y \to aY \mid \varepsilon$



16.

	A	B
→F 1	1	2
2	1	1



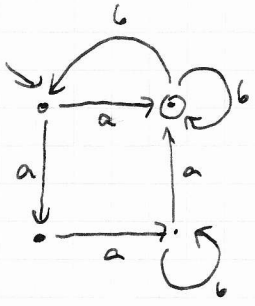
5) $((a|b)(a|b))^*$

d

	a	b
→1	2	2
F 2		1

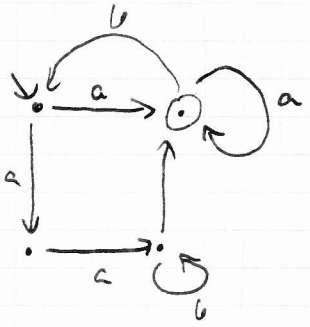
2) $(a|b)(ba|bb)^*$

2.b



3) $(aab^*a|a)(b^*|ba|baab^*$

d



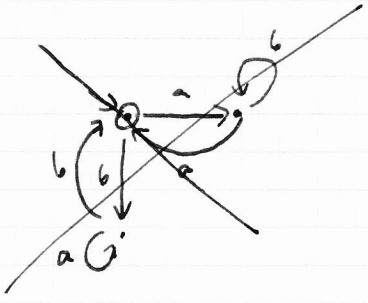
← tu brak bl'ędu! (b)

5)
$(a|aab^*a)(a^*|ba|baab^*a)^*$

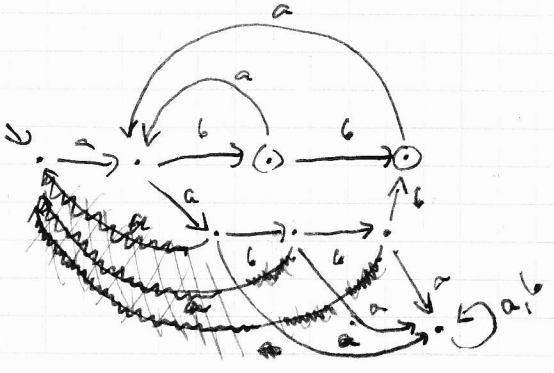
3.

.. $(ab^*a)^*|(ba^*b)^*$

Czy da się łatwo zauważyć?

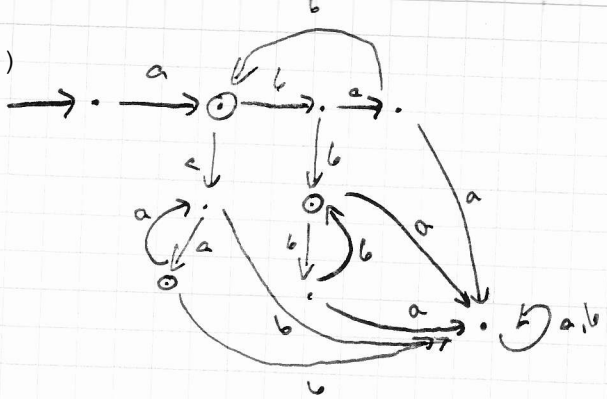


.... $(a|abb|aabb)^+$

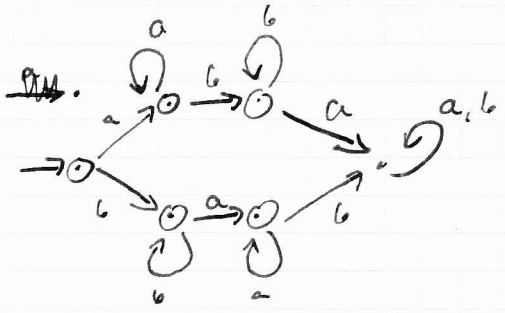


.....

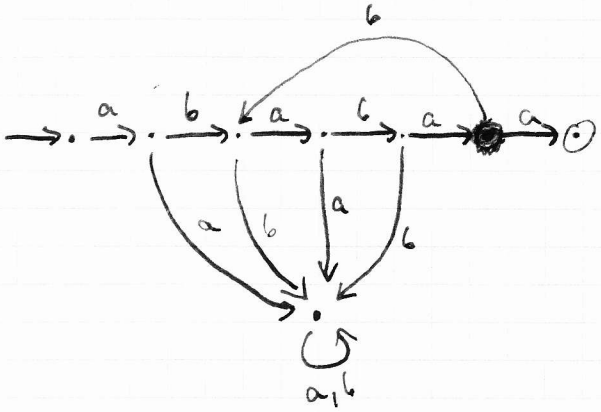
a(bab)*|a((aa)*|(bb)*)



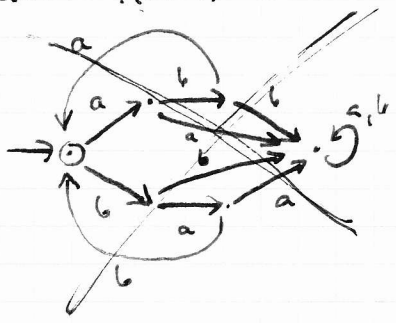
4. .. $a^*b^* \mid b^*a^*$



.... $a(b+ab+a)^+a$ To nie jest wyrażenie regularne!

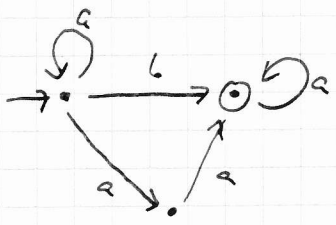


..... $(ab+a)^* \mid (b+ab)^*$



5.

a)

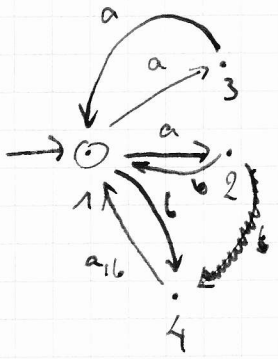


~~$a^* \mid a^* b a^* \mid a^* (aa) a^*$~~

$a^* (b \mid aa) a^*$

b)

		a	b
→ F	1	2,3	4
	2	-	1
	3	1	-
	4	1	1

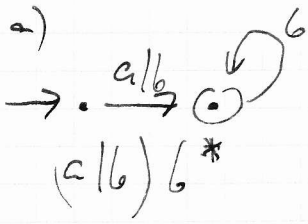


$((aa)^* \mid (ab)^* \mid (ba)^* \mid (bb)^*)^*$

$(a^*(ab)^* \mid b^*(ab)^*)^*$

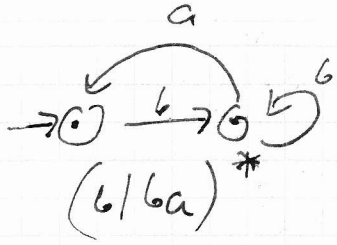
$((a \mid b)(a \mid b))^*$

1. a)



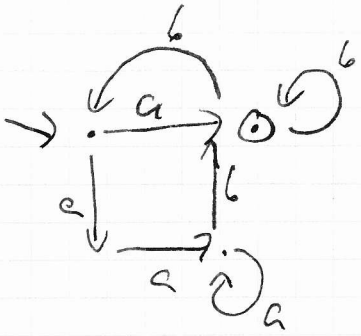
$(a|b)\,b^*$

c)



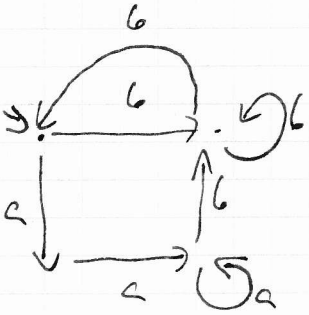
$(b|ba)^*$

2, c)



$(a|aaa^*b)\,(ba|b^*|baaa^*b)^*$

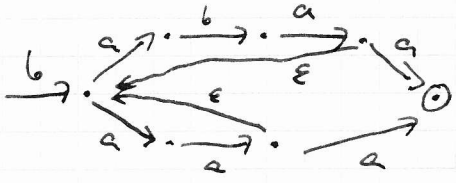
e)



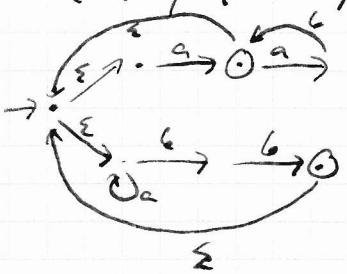
$(b|aaa^*b)\,(b^*|bb|baaa^*b)^*$

3.

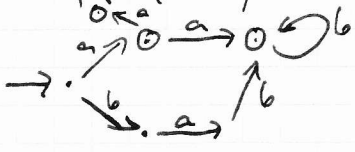
3° $b(aba|aa)^+a$



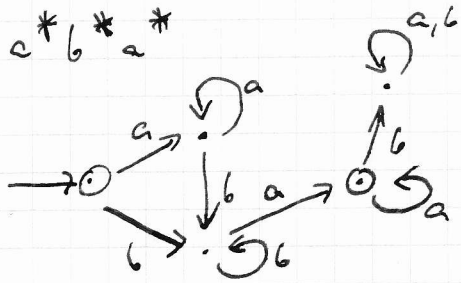
1° $(a(ab)^*|a^*bb)^*$



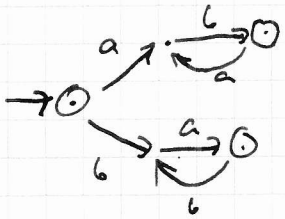
5°° $aa|(a|ba)\,b^*$



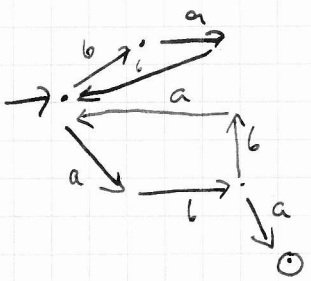
4. 1) $a^* b^* a^*$



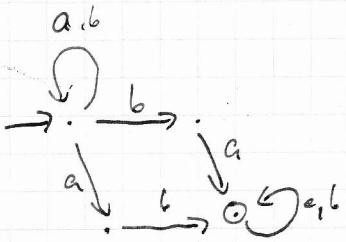
3) $(ab)^* \mid (ba)^*$



5) $(baab \mid abba)^* aba$

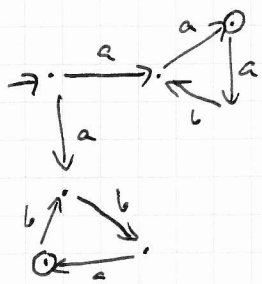


5.



$(aa|b)^*(ab|ba)(aa|b)^*$

6. 3)



$aba(bba)^* \mid aa(aba)^*$

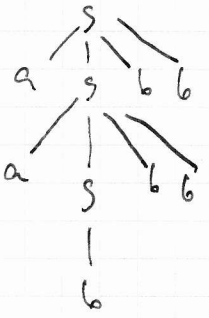
AUG ĆW 13.04.2011

języki bezkontekstowe

aa bbbbb

2. $\{a^i b^{2i+1} : i \geq 0\}$

$S \to aSbb \mid b$



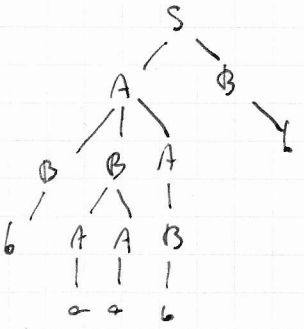
4. $S \to AB \mid BA$

$A \to a \mid BBA$

$B \to b \mid ABA$

$S \to AB \to BBABA \to bBABA \to$

$\to bAABABA \to baababa$



jednoznaczny

7. 4° $\{a^n b^k : 2n \geq 3k\}$

$S \to aaaSbb \mid aaSb \mid aS \mid \varepsilon$ jednoznaczne

10° $\{a^n b^m : n \geq 2m \geq 0\}$

$S \to aaSb \mid aS \mid \varepsilon$

8° $\{a^i b^j c^k : i+2 \cdot j = k\}$

~~$S \to$ [illegible]~~

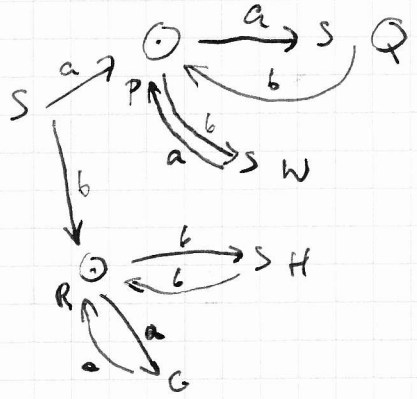
$S \to X \mid aSc \mid \varepsilon$

$X \to bXcc \mid \varepsilon$

11.

2°

$(a(ab|ba)^*)|(b(bb|aa)^*)$



$S \to SaP \mid bR$

$P \to SaQ \mid bW \mid \varepsilon$

$Q \to SbH \mid aG \mid \varepsilon$

$R \to SbP$

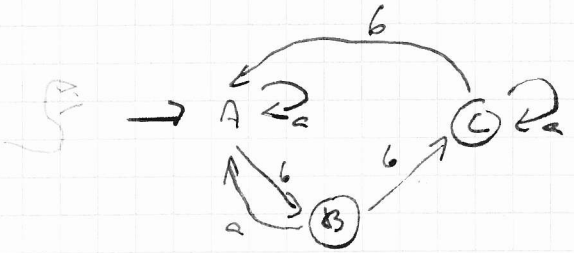
$W \to SaP$

$H \to SGR$

$G \to SaR$

4°

	a	b
→A	AB	
FB	AC	
FC	CA	



$S \to A$

$A \to SaA \mid bB$

$B \to SaA \mid bC \mid \varepsilon$

$C \to bA \mid aC \mid \varepsilon$

3. $\{a^i b^{i+2j} c^j : i,j \geq 0\}$

$a^i b^i \; b^{2j} c^j$

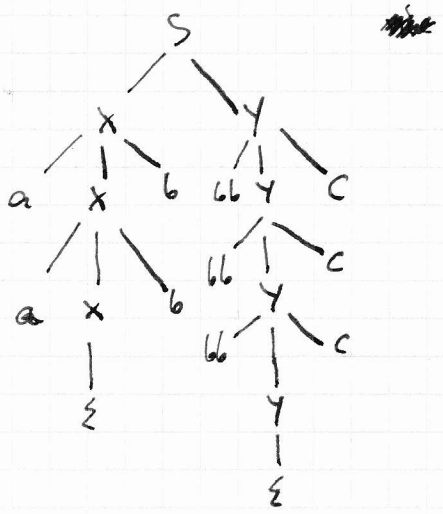
X Y

aabb bb bb bbcc cc

$S \to XY$

$X \to aXb \mid \varepsilon$

$Y \to bbYc \mid \varepsilon$



5. $S \to A \mid B \mid AB$

$A \to BA \mid aaA \mid abaA \mid aa$

$B \to abbB \mid \varepsilon$

niejednoznaczna

$S \to AB \to BAB \to BA \to abbBaa \to$

$\to abbabbaa$

abbabbaa

7. 5° $\{a^i b^j c^i\}$

$S \to aSc \mid X \mid \varepsilon$

$X \to bX \mid \varepsilon$

7° $\{a^i b^i a^j b^j\}$

$S \to AB$

$A \to aAb \mid \varepsilon$

$B \to aBb \mid \varepsilon$

n°

$\{a^m b^n c^k d^l : n+m = k+b\}$

$S \to aSd \mid X \mid Y \mid Z \mid \varepsilon$

$X \to aXc \mid Z \mid \varepsilon$

$Y \to bYd \mid Z \mid \varepsilon$

$Z \to bZc \mid Z \mid \varepsilon$

m)

1° $\underset{A}{a}\underbrace{(a \mid b)^*}_{B}\underbrace{aa}_{C}$

$S \to ABC$

$A \to a$

$B \to aB \mid bB \mid \varepsilon$

$C \to aa$

jednoznaczna

to nie jest liniowe rozwiązanie!

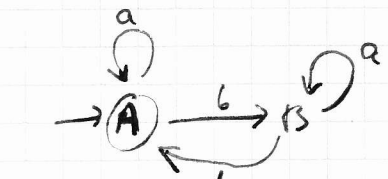
3°

	a	b
→FA	A	B
B	B	A

$S \to A$

$A \to aA \mid bB \mid \varepsilon$

$B \to aB \mid bA$



dowolna gramatyka

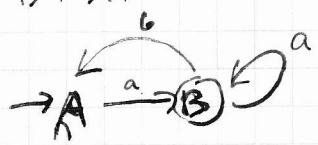
$S \to AB \mid A \mid \varepsilon$

$A \to aA \mid bbA \mid \varepsilon$

$B \to aB \mid \varepsilon$

5°

	a	b
→A	B	-
FB	B	A



$S \to A$

$A \to aB$

$B \to aB \mid bA \mid \varepsilon$

AUG (automaty stosowe)

1.

$(s, x) \xrightarrow{(} (s,)x)$ dla $x = \bot,)$

$(s, x) \xrightarrow{[} (s,]x)$ dla $x = \bot,]$

$(s, x) \xrightarrow{x} (s, \varepsilon)$ dla $x = \},]$ ← zdejmowanie ze stosu

$(s, \bot) \xrightarrow{\varepsilon} (s, \varepsilon)$ ← ostatnie przejście, zdjęcie pierwiastka

$(s, x) \xrightarrow{\{} (p, \}x)$ dla $x = \bot, \},]$

$(p, x) \xrightarrow{[} (p,]x)$ dla $x =]$

$(p, \}) \xrightarrow{\}} (s, \varepsilon)$

$(p,]) \xrightarrow{]} (p, \varepsilon)$

$(s, x) \xrightarrow{(} (s,)x)$ $x = \bot, \},],)$

$(s, x) \xrightarrow{[} (s,]x)$ $x = \bot, \},]$

$(s, x) \xrightarrow{\{} (p, \}x)$ $x = \bot, \}$

$(s, x) \xrightarrow{x} (s, \varepsilon)$ $x = \},],)$

$(s, \bot) \xrightarrow{\varepsilon} (s, \varepsilon)$

$(p, x) \xrightarrow{\{} (p, \}x)$ $x = \}$

$(p, x) \xrightarrow{[} (p,]x)$ $x = \},]$

$(p,]) \xrightarrow{]} (p, \varepsilon)$

$(p, \}) \xrightarrow{\}} (s, \varepsilon)$

3. $a^i b^{2i}$

Trudniejszy

$(s, S) \xrightarrow{\varepsilon} (s, aSbb)$
$(s, S) \xrightarrow{\varepsilon} (s, \varepsilon)$
$(s, a) \xrightarrow{a} (s, \varepsilon)$
$(s, b) \xrightarrow{b} (s, \varepsilon)$

nie deterministyczny (trudniejszy)

$(s, \bot) \xrightarrow{a} (s, AA\bot)$
$(s, A) \xrightarrow{a} (s, AAA)$
$(s, A) \xrightarrow{b} (q, \varepsilon)$
$(q, A) \xrightarrow{b} (q, \varepsilon)$
$(q, \bot) \xrightarrow{b} (q, \varepsilon)$
$(q, \bot) \xrightarrow{\varepsilon} (q, \varepsilon)$

5. $w \# a(w) >= 2 \# b(w)$

Kaba litera a o dwie a ma być zamiennie z trzy ciężka

$(s, \bot) \xrightarrow{a} (s, A\bot)$ $(s, \bot) \xrightarrow{b} (s, BB\bot)$
$(s, A) \xrightarrow{a} (s, AA)$ $(s, B) \xrightarrow{b} (s, BBB)$
$(s, A) \xrightarrow{b} (a, \varepsilon)$ $(s, B) \xrightarrow{a} (s, \varepsilon)$

$(q, A) \xrightarrow{\varepsilon} (s, \varepsilon)$
$(q, \bot) \xrightarrow{\varepsilon} (s, B\bot)$
$(s, A) \xrightarrow{\varepsilon} (s, \varepsilon)$
$(s, \bot) \xrightarrow{\varepsilon} (s, \varepsilon)$

~~Wyrażenia~~ a, b, c

2. palindrom $\{a, b, c\}$

$(s, \perp) \xrightarrow{a} (s, A\perp)$
$(s, \perp) \xrightarrow{b} (s, B\perp)$
$(s, \perp) \xrightarrow{c} (s, C\perp)$

$(s, A) \xrightarrow{x} (s, xA)$
$(s, B) \xrightarrow{x} (s, xB)$
$(s, C) \xrightarrow{x} (s, xC)$
$x = a, b, c$
$X = A, B, C$

$(s, x) \xrightarrow{\varepsilon} (p, x)$
$(s, x) \xrightarrow{y} (p, x)$ dla $y = a, b, c$
$(p, A) \xrightarrow{a} (p, \varepsilon)$
$(p, B) \xrightarrow{b} (p, \varepsilon)$
$(p, C) \xrightarrow{c} (p, \varepsilon)$
$(p, \perp) \xrightarrow{\varepsilon} (p, \varepsilon)$
$(s, \perp) \xrightarrow{\varepsilon} (p, \perp)$
$(s, \perp) \xrightarrow{\varepsilon} (q, \perp)$
$(q, \perp) \xrightarrow{\varepsilon} (q, \varepsilon)$

Praca dom
AUG

Zad 1

$(S, x) \xrightarrow{\{} (S, \{x)$ dla $x = \bot, \{$

$(S, x) \xrightarrow{[} (S,]x)$ dla $x = \bot] \{$

$(S, x) \xrightarrow{(} (S,)x)$ dla $x = \bot] \{)$

$(S, x) \xrightarrow{\varepsilon} (S, \varepsilon)$ dla $x = \},],)$

$(S, \bot) \xrightarrow{\varepsilon} (S, \varepsilon)$

Zad 2

aSa | bSb | a | b | ε

$(s, S) \xrightarrow{\varepsilon} (s, aSa)$

$(s, S) \xrightarrow{\varepsilon} (s, bSb)$

$(s, S) \xrightarrow{\varepsilon} (s, a)$

$(s, S) \xrightarrow{\varepsilon} (s, b)$

$(s, S) \xrightarrow{\varepsilon} (s, \varepsilon)$

$(s, a) \xrightarrow{a} (s, \varepsilon)$

$(s, b) \xrightarrow{b} (s, \varepsilon)$

babbbab

b
a s s b
b s s b b b
s a a a a a a a
> b b b b b b b b b >

Zad 3

$(s, x) \xrightarrow{[} (S,]x)$ dla $x =], \bot$

$(s,]) \xrightarrow{]} (s, \varepsilon)$

$(s, \bot) \xrightarrow{\varepsilon} (s, \varepsilon)$

S → [AS | ε

A → [AA |]

lub

S → [S]S | ε

np. [[][]]

S → [AS → [[AAS → [[][AAS →
→ [[][]]S → [[][]]

AUG

Kuba Przybyłowicz

Zad 1

a) $L = \{w \in \{a,b\}^* : w \text{ nie zawiera podsłowa } bb\}$

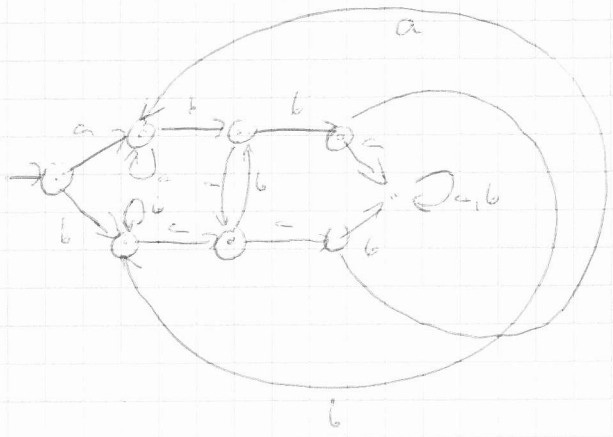
$$(a|ba)^*(b|\varepsilon)$$

b) $L = \{w \in \{a,b\}^* : w \text{ nie zawiera żadnego z słów } bba, abb\}$

$$(a|b)^*(baa|aab)(a|b)^*$$

Zad 2

a) $L = \{w \in \{a,b\}^* \text{ bez } bab \text{ i } abba\}$



Zad 3

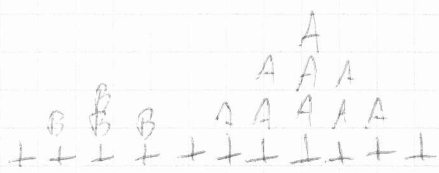
$w = b^i a^j b^k \quad i+k=j$

$(s,\bot) \xrightarrow{b} (s,B)$

$(s,X) \xrightarrow{b} (s,B)$ dla $X = \bot, b$

- $(s,\bot) \xrightarrow{\varepsilon} (p,\bot)$
- $(s,\bot) \xrightarrow{b} (s,B\bot)$
- $(s,B) \xrightarrow{b} (s,BB)$
+ $(p,B) \xrightarrow{a} (p,\varepsilon)$
- $(p,\bot) \xrightarrow{a} (p,A\bot)$
- $(p,A) \xrightarrow{a} (p,AA)$
- $(p,A) \xrightarrow{b} (q,\varepsilon)$
- $(q,A) \xrightarrow{b} (q,\varepsilon)$
- $(s,B) \xrightarrow{a} (p,\varepsilon)$
- $(q,\bot) \xrightarrow{\varepsilon} (q,\varepsilon)$
- $(p,\bot) \xrightarrow{\varepsilon} (q,\bot)$

bb aa aaa bbb



15 [illegible] nie ma [illegible]